fringe patterns by the application of wavelet filters in convolution processors. Journal of Electronic Imaging 10:228-233

The Virtual Fringe Projection System (VFPS) and Neural Networks

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1 Introduction

Optical measurement systems like fringe projection systems (FPS) are complex systems with a great number of parameters influencing the measurement uncertainty. Pure experimental investigations indeed are unable to determine the influence of the different parameters on the measurement results. The virtual fringe projection system was developed for this purpose. It gives the possibility to control parameters individually and independently from other parameters [1]. The VFPS is a computer simulation of a fringe projection system and mainly developed to investigate different calibration methods.

While several black-box calibration methods are shown in [1], neural networks is the main subject of this paper.

2 Neural Networks

Many different kinds of artificial neural networks have been developed, thereof backpropagation networks are probably the most well-known. Most neural networks can be considered simply as a nonlinear mapping between the input space and the output space. When a neural network is provided with a set of training data, it will be able to respond with the correct answer after a learning phase—at least below a given error margin. Neural networks’ generalization ability means the effect of a nonlinear approximation on new input data.
Backpropagation and radial basis function (RBF) networks are appropriate for approximation of functions [2]. Backpropagation networks construct global approximations to the nonlinear input-output mapping, whereas RBF networks construct local approximations. The calibration process of a VFPS means mathematically the determination of the nonlinear calibration function \( f \). The function \( f \) describes the relationship between the image coordinate system, consisting of the camera pixels \((i, j)\), as well as the phase value \( \phi \), and the object coordinate system \((X, Y, Z)\), i.e.

\[
(X, Y, Z) = f(i, j, \phi)
\]  

(1)

Experimental investigations of backpropagation networks provide bad results, therefore only RBF networks will be considered. RBF networks consist of one hidden layer and one output layer. The hidden layer consists of radial basis neurons. The output value \( o_i \) of neuron \( i \) is

\[
o_i = h\left(\|w_i - x\|\right)
\]

(2)

with \( h \) as a radial basis function like the Gaussian bell-shaped curve

\[
h(r) = \exp\left(-\alpha r^2\right), \quad \alpha > 0
\]

(3)

and input vector \( x \) and weight vector \( w_i \).

Because the Gaussian curve is a localized function, i.e. \( h(r) \to 0 \) as \( r \to \infty \), each RBF neuron approximates locally. The parameter \( \alpha \) determines the radius of an area in the input space to which each neuron responds.

The output layer is linear mapping from the hidden space into the output space. Each output neuron delivers directly an output value. The number of hidden neurons is normally much greater than the number of input signals. In the case of the calibration task, there are three input signals (image coordinates) and three output signals (object coordinates).

### 3 Simulation and results

With the VFPS it's possible to evaluate exclusively the error of a RBF network, due to the fact that it allows the investigation of the calibration method under ideal conditions, independently from other influences [1]. The dimension of the used measuring volume is 120 x 90 x 40 (the unit is set to one) in the VFPS. For the calibration process 20 x 15 x 10 (in X, Y and Z direction) calibration points \( p \), uniformly distributed in the measuring volume, is used. With this setup, the RBF network is calculated using the VFPS. For this purpose the phase value \( \phi \) for all calibration points \( p \) will be calculated first (via a direct projection on the projector plane and calculation of the corresponding phase value at this point). Subsequently, the points \( p \) will be projected on the image plane of the camera, delivering the corresponding image coordinates \((i, j)\). Now the calibration function \( f \) can be determined with the aid of all known values \((i, j, \phi)\) and \((X, Y, Z)\).

In order to determine the resulting error of the RBF network 10,000 points are randomly generated in the measuring volume and "measured" with the VFPS. The corresponding object coordinates of these points are then calculated by means of the previously determined RBF network.

The resulting standard deviation of the RBF network was 9.8 x 10^{-4}. For the same configuration a polynomial method (method C in [1]) yielded a standard deviation of 9.2 x 10^{-4}.

Figure 1 shows the 3D gray-coded error map of the RBF network. Three section planes with the measuring volume show the deviation of the calculated values from the correct values. To show border effects, the volume is extended by 10% in each direction compared to the calibration process.

Figure 2 shows additionally a similar diagram for the polynomial method mentioned above.

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Fig. 1. Gray coded 3D error map for the RBF network, three section planes with the measurement volume visible
4 Conclusion

The RBF network and the polynomial method investigated previously produce comparable results. So far no evident arguments can be found to prefer it as calibration method. Nevertheless, further investigations have to show the effects of the influence of different disturbances (like noise) on different methods.

5 Acknowledgment

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6 References