Sensitivity Analysis of a Two-Lens System for Positioning Feedback

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Precise alignment of optical components is crucial for the assembly of optical system. Misaligned components (due to positioning tolerances) lead to a distorted wavefront that deviates from a desired wavefront given by an initial nominal design. Inferring the current poses of optical components solely from detector measurements is a difficult but necessary task in order to calculate corrections for feedback control without using external measurement devices. In this paper, we aim at deducing the lens poses of a two-lens system solely from wavefront measurements. The wavefront can be decomposed into a finite sequence of polynomials which is weighted by coefficients. Since pose deviations directly lead to deviations in the wavefront (and therefore in the wavefront coefficients), a sensitivity matrix can be established. The choice of lens poses and wavefront coefficients determines the sensitivity matrix which is often ill-conditioned and may potentially suffer from rank loss. We conduct a sensitivity analysis and show how it can be exploited to generate a linear estimate of the optical component poses. This estimate can then be used to close the feedback loop. Validation is conducted by simulation of an optical system consisting of a laser, two bi-convex lenses, and a wavefront sensor.

1 Introduction

Assembly of optical systems relies on the precise placement of its components [1]. Due to strict tolerance requirements in optical systems (especially those based on interferometry), it is crucial to precisely align the components in order to guarantee the functionality of the optical system. In this paper, alignment is achieved by measuring the wavefront at the detector, inferring an optical component pose estimate, and utilize this to close the feedback loop.

Wavefronts are often represented by so-called (normalized) Zernike polynomials Z_n^m. These polynomials provide an orthogonal (orthonormal) set of scalar functions over the circular domain and are defined as

$$Z_n^m = \begin{cases} N_n^m R_n^m(\rho)\sin(m\theta) & \text{for } m < 0 \\ N_n^m R_n^m(\rho)\cos(m\theta) & \text{for } m \geq 0 \end{cases}, \qquad \text{where} \tag{1}$$

$$N_n^m = \sqrt{\frac{2(n+1)}{1+\delta_{m0}}}, \delta_{m0} = \begin{cases} 1 & \text{for } m = 0 \\ 0 & \text{for } m \neq 0. \end{cases}, \text{ and } R_n^m(\rho) = \sum_{k=0}^{(n-|m|)/2} \frac{(-1)^k (n-k)!}{k! \left(\frac{n+|m|}{2} - k\right)! \left(\frac{n-|m|}{2} - k\right)!} \rho^{n-2k}.$$

Therein, N_n^m is a normalization factor, R_n^m a radial polynomial, $0 \leq \rho \leq 1$ the normalized radial distance, and $0 \leq \theta \leq 2\pi$ the azimuthal angle. By mapping these polynomials to a single index (e.g. by using Noll's sequence [2]), a linear expansion of a wavefront Φ can be realized by $\Phi = \sum_{j=1}^{n_z} z_j Z_j$. The Zernike coefficients z_j can be provided by a wavefront sensor.

2 Sensitivity Analysis and Linear Feedback Control of a Two-Lens System

An optical system is typically comprised of a light source, optical components, and a sensor (see Fig. 1). A measurement function $\mathbf{z} = h(\mathbf{x})$ maps component poses $\mathbf{x}$ to a wavefront which can be represented by Zernike polynomials. The corresponding coefficients $\mathbf{z}$ are then obtained by a wavefront sensor. Linearizing the function h w.r.t. the component poses yields

$$\frac{d\mathbf{z}}{d\mathbf{x}} = \frac{dh(\mathbf{x})}{d\mathbf{x}} \approx \mathbf{S} \qquad \Rightarrow \qquad d\mathbf{z} = \mathbf{S} \cdot d\mathbf{x}, \tag{2}$$

where $\mathbf{S}$ is the Jacobian (also: sensitivity matrix). An estimate of the change of component poses can be obtained by $d\mathbf{x} = \mathbf{S}^\dagger d\mathbf{z}$, where $\mathbf{S}^\dagger$ is the pseudoinverse of the sensitivity matrix.

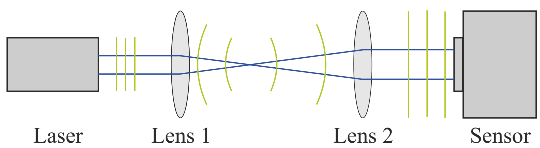

Fig. 1: Two-lens system emitting a wavefront (green) from a light source that passes through optical elements ending at a sensor.

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The pose estimate of the optical components can then be utilized to realize feedback control (see Fig. 2). In real world applications, the positioning system is subject to uncertainty $\mathbf{d}_x \sim \mathcal{N}(0, \mathbf{\Sigma}_x)$ and the wavefront sensor is subject to noise $\mathbf{d}_z \sim \mathcal{N}(0, \mathbf{\Sigma}_z)$. Assuming additive noise, the estimate computes as

$$\mathbf{x}^* = \mathbf{S}^\dagger \mathbf{z} + \mathbf{S}^\dagger \mathbf{d}_z, \tag{3}$$

where $\mathbf{x}^*$ is the optical pose estimate.

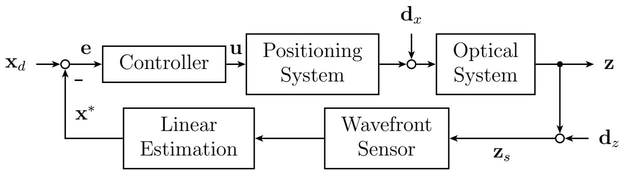
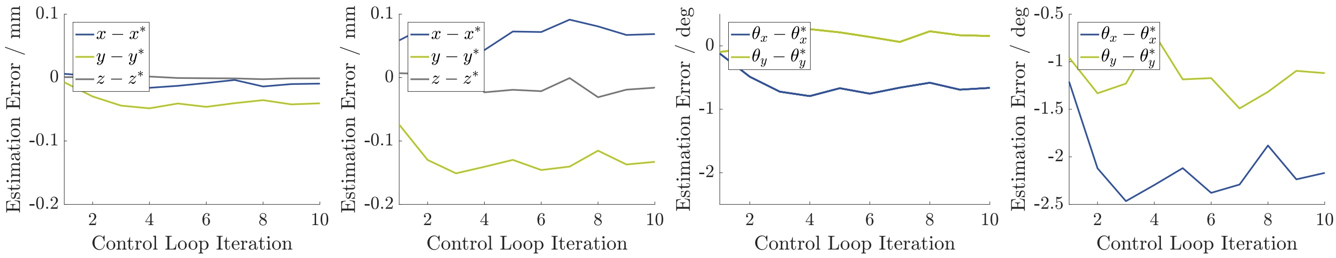


Fig. 2: Control loop for optical component alignment with linear wavefront-based pose estimation.

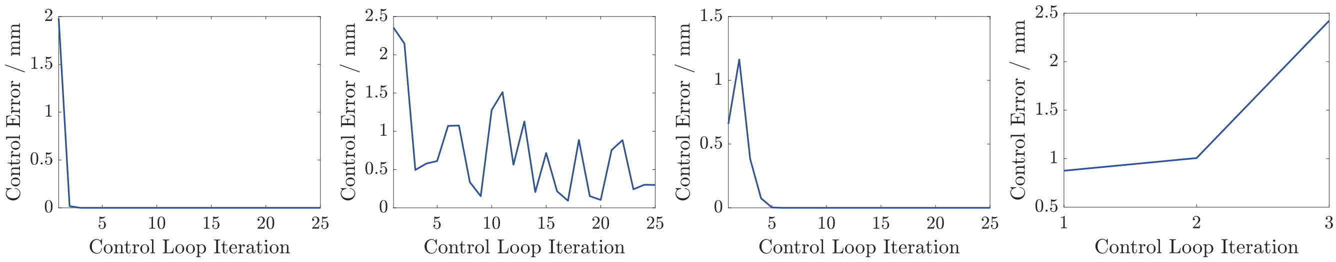
3 Simulation Results

For the simulation, we use a system comprised of two bi-convex lenses (see Fig. 1) with a focal length of $f_1 = 50\,\mathrm{mm}$ and $f_2 = 100\,\mathrm{mm}$, respectively. A nominal distance of $150\,\mathrm{mm}$ between the lenses results in a planar wavefront. Fig. 3 shows how the standard deviation of the sensor noise influences the estimation error. Due to the ill-conditioned sensitivity matrix ($\kappa(\mathbf{S}) = 9.82 \cdot 10^4$), increasing the sensor noise σ_z directly maps to an increasing estimation error, see (3). Fig. 4 shows the control error with increasing uncertainty of the positioning system. Simulation results show that the two-lens system is robust in the despace direction while slight misalignment in tilt direction can lead to instability and divergence of the control loop.



(a) $\sigma_z = 10^{-5}$ **(b)** $\sigma_z = 10^{-4}$ **(c)** $\sigma_z = 10^{-5}$ **(d)** $\sigma_z = 10^{-4}$

Fig. 3: Estimation error of optical component poses with increasing sensor noise for translation (a,b) and rotation (c,d).



(a) Despace, $\sigma_x = 0$ **(b)** Despace, $\sigma_x = 10^{-1}$ **(c)** Tilt, $\sigma_x = 0$ **(d)** Tilt, $\sigma_x = 10^{-1}$

Fig. 4: Increasing position uncertainty in despace direction (a,b) and tilt direction (c,d).

4 Conclusion

In this paper, sensitivity analysis of a two-lens system has been presented and the performance of using a linear estimate for feedback control evaluated. Due to the low computational complexity of this approach, it is suited for fast online in-process control. However, the pseudoinverse of the Jacobian is often ill-conditioned and therefore amplifies sensor noise. This may lead to instability and divergence of the control loop. Furthermore, the position uncertainty of the positioning system may also lead to instability. However, sensitivity analysis can be utilized as a tool to identify sensitive and robust directions for the positioning uncertainty. This information can be used in order to design positioning systems or evaluate the tolerance specifications when choosing such systems for optical assembly.

References

[1] C. Schindlbeck, C. Pape, and E. Reithmeier, Optics Express **26**, 10669–10681 (2018).
[2] R. J. Noll, J. Opt. Soc. Am. **66**(3), 207–211 (1976).