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АВТОМАТИКА И ИНФОРМАТИКА '02

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PSEUDO INDIRECT STABILIZING ADAPTIVE CONTROLLER DESIGN FOR A LINEARIZED MODEL OF AN INVERTED PENDULUM AROUND VERTICAL POSITION

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Abstract: This paper presents some partial investigations about design of a adaptive stabilizing controller for the model of an inverted pendulum obtained with classical linearization around vertical position of a complete nonlinear model. Using Lyapunov redesign technique together with LaSalle invariance principle, adaptive laws of proposed controller are obtained. Applying well known Barbalat's like lemmas the convergence of adaptive laws is proven and in this way whole stability of the scheme is proved. Numerical simulations are also carried out for confirming the performance and robustness properties of the design.

Keywords: adaptive stabilization, Lyapunov redesign, stability analysis, convergence analysis; numerical simulations

1. INTRODUCTION

The problem considered in the recent paper is the adaptive stabilization of an inverted pendulum, exploiting a continuous-time model obtained via linearization around upright position of a nonlinear model, derived with taking into account the spin torque, if it is eventually acting on the pendulum, [1]. A similar problem is considered in [1], but using a nonlinear model.

There exists amount of developments concerning stabilization and control of inverted pendulum, see for example [15] and references therein. In the cited paper discrete H_∞ control is employed for a linearized discrete-time model of a triple inverted pendulum about upright position with zero input.

One of the first pioneer investigations on adaptive stabilization of linear systems in state space is Morse's universal stabilizer, [2]. The main assumption in this work is the relative degree of the plant to be not greater than two. A brief state of the art is given in [3]. This paper extends the Morse's results for relative degree r, r+1 and r+2. In the cited paper is also mentioned another important result of Morse, [4], in which reference model is used in the design. We also have to mention the manuscripts on adaptive control, [5, 7, 13, 14] where the problem for regulation and control in case of using model reference, pole placement, etc., are studied in details.

2. MATHEMATICAL PRELIMINARIES

We will cite an important theorem proven in [11], but first, we give the definition for quadratic stability. Consider the system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t); \\ y(t) &= Cx(t) \end{aligned} \qquad (\Sigma)$$

Where $x(t) \in R^n$ is the *plant state*, $u(t) \in R^m$ is the *control input*, and $y(t) \in R^q$ is the *measured output*. It is assumed that the matrix C is of full rank. Applying a direct output feedback control law of the form $u(t) = p(y(t))$ to the system (Σ) results in a closed-loop system described by the state equation.

$$\dot{x}(t) = Ax(t) + Bp(Cx(t)) \qquad (\Sigma_{cl})$$

Definition: *The nonlinear system is said to be quadratically stable if the origin $x = 0$ is an equilibrium point and, furthermore, there exists a positive-definite symmetric matrix P and a constant $\alpha > 0$ such that the following condition holds. The Lyapunov derivative corresponding to the Lyapunov function $V(x) = x'Px$ and the system satisfies the bound*

$$\begin{aligned} L(x) &\triangleq 2x'P\left[Ax + Bp(Cx)\right] \\ &\le -\alpha \|x\|^2 \end{aligned}$$

For all $x \in R^n$.

Next remark made in [11] is of great importance.

Remark: If the system is quadratic ally stable and the function $p(.)$ is continuous, then it is straightforward to verify that the system will be uniformly and asymptotically stable.

Theorem: *Consider a linear time-invariant system of the form (Σ) and suppose that there exists a direct output feedback control law $u(t) = p(y(t))$ such that he resulting closed-loop system is quadratic ally stable. Then there exists a linear direct output feedback control law $u(t) = Ky(t)$ such that the resulting closed-loop system is asymptotically stable; that is, all of eigenvalues of $A + BKC$ lie in the strict left half plane.*

We introduce also two important lemmas useful for the development to follow, found in [5]

Lemma 1. (Barbalat's Lemma). *If* $\lim_{t\to\infty}\int_0^t f(\tau)d\tau$ *exists and is finite, and* $f(t)$ *is uniformly continuous function, then* $\lim_{t\to\infty} f(t)=0$.

The following corollary follows directly from the above lemma, [5].

Corollary. If $f,\dot{f}\in L_\infty$ and $f\in L_p$ for some $p\in[1,\infty)$, then $f(t)\to 0$ as $t\to\infty$.

Lemma 2. *The following is true for scalar-valued functions:*

(i) A function $f(t)$ *that is bounded from below and is non-increasing has a limit as* $t\to\infty$.

(ii) Consider the nonnegative scalar functions $f(t)$, $g(t)$ *defined for*

(iii) all $t\geq 0$. *If* $f(t)\leq g(t)$ $\forall t\geq 0$ *and* $g\in L_p$

Then $f\in L_p$ *for all* $p\in[1,\infty)$

3. STATEMENT OF THE PROBLEM

We consider multivariable linearized model of an inverted pendulum around upright position described in state space form, i.e.

$$\begin{bmatrix}\Delta\dot{\psi}(t)\\ \Delta\ddot{\psi}(t)\\ \Delta\dot{\upsilon}(t)\\ \Delta\ddot{\upsilon}(t)\end{bmatrix}=\frac{1}{\Theta}\begin{bmatrix}\begin{pmatrix}0&1\\g&0\end{pmatrix} & \begin{pmatrix}0&0\\0&-\alpha\omega_0(t)\end{pmatrix}\\ \begin{pmatrix}0&0\\0&\omega_0(t)\end{pmatrix} & \begin{pmatrix}0&1\\g&0\end{pmatrix}\end{bmatrix}\begin{bmatrix}\Delta\psi(t)\\ \Delta\dot{\psi}(t)\\ \Delta\upsilon(t)\\ \Delta\dot{\upsilon}(t)\end{bmatrix} + \frac{1}{\Theta}\begin{bmatrix}0&0\\1&0\\0&0\\0&-1\end{bmatrix}\begin{bmatrix}\nabla u_1(t)\\ \nabla u_2(t)\end{bmatrix} \quad (1)$$

This model is obtained from the multivariable nonlinear model via linearization around vertical position, [1], taking into account a spin torque, if any, acting on it. In (1) ψ and ϑ are the two position angles of the pendulum in appropriate coordinate system, [1]; g is the acceleration via the gravity, ω_0 is the spin torque, α is a small positive constant, Θ is positive constant including mass, length, and inertia moment of the pendulum. Δu_1, Δu_2 - are control variables, acting on the platform with the pendulum, to keep it in vertical position.

As is seen from (1) the state matrices are unstable, but they are the same for both angles. The interconnecting matrices however, are different.

The vector equations of the plant, (1) can be rewritten using more obvious designations as follows

$$\begin{bmatrix}\dot{x}_1(t)\\ \ddot{x}_1(t)\\ \dot{x}_2(t)\\ \ddot{x}_2(t)\end{bmatrix}=\begin{bmatrix}\begin{pmatrix}0&1\\a_{21}&0\end{pmatrix} & \begin{pmatrix}0&0\\0&a_{24}(t)\end{pmatrix}\\ \begin{pmatrix}0&0\\0&a_{42}(t)\end{pmatrix} & \begin{pmatrix}0&1\\a_{43}&0\end{pmatrix}\end{bmatrix}\begin{bmatrix}x_1(t)\\ \dot{x}_1(t)\\ x_2(t)\\ \dot{x}_2(t)\end{bmatrix} + \begin{bmatrix}\begin{pmatrix}0&0\\b_{21}&0\end{pmatrix}\\ \begin{pmatrix}0&0\\0&b_{42}\end{pmatrix}\end{bmatrix}\begin{bmatrix}u_1(t)\\ u_2(t)\end{bmatrix} \quad (2)$$

The meaning of symbols in (2) is clear in comparing with (1) and will be given later on in the part of numerical simulations.

The purpose of the presented paper is to synthesise adaptive stabilizing control law for the plant described by differential equation (1), (2), to prove analytically stability and convergence of the closed loop system and with numerical simulations to demonstrate the performance and certain robustness possibilities of the derived adaptive stabilizing scheme.

4. SYNTHESIS OF THE ADAPTIVE STABILIZING CONTROLLER

We choose the following adaptive structure for stabilizing the plant described by (1) and (2)

$$u_1(t)=-\theta_{21}(t)x_1(t)+\theta_{22}(t)\dot{x}_1(t)+\theta_{24}(t)\dot{x}_2(t) \quad (3)$$
$$u_2(t)=-\theta_{42}(t)\dot{x}_1(t)+\theta_{43}(t)x_2(t)+\theta_{44}(t)\dot{x}_2(t)$$

Six expressions in (3), $\theta_{21}(t)$, $\theta_{22}(t)$, $\theta_{24}(t)$ and $\theta_{42}(t)$, $\theta_{43}(t)$, $\theta_{44}(t)$ are the adaptive laws which will be obtained in the next section. In the sequel, for clarity of presentation, we will omit the time index, where it is possible.

Substituting from (3) into (2) we obtain the following closed loop scheme

$$\begin{bmatrix}\dot{x}_1\\ \ddot{x}_1\\ \dot{x}_2\\ \ddot{x}_2\end{bmatrix}=\begin{bmatrix}\begin{pmatrix}0&1\\a_{21}-b_{21}\theta_{21}&b_{21}\theta_{22}\end{pmatrix} & \begin{pmatrix}0&0\\0&a_{24}(t)+b_{21}\theta_{24}\end{pmatrix}\\ \begin{pmatrix}0&0\\0&a_{42}(t)+b_{42}\theta_{42}\end{pmatrix} & \begin{pmatrix}0&1\\a_{43}-b_{42}\theta_{43}&b_{42}\theta_{44}\end{pmatrix}\end{bmatrix}\begin{bmatrix}x_1\\ \dot{x}_1\\ x_2\\ \dot{x}_2\end{bmatrix} \quad (4)$$

5. LYAPUNOV REDESIGN

To obtain the expressions for adaptive laws and to prove stability of the scheme (4) we propose the following structure for potential Lyapunov function

$$\begin{aligned}V(t)=&(x_1^2+\dot{x}_1^2+x_2^2+\dot{x}_2^2)/2\\ &+\frac{1}{\gamma_{21}}\tilde{\theta}_{21}\operatorname{sgn}(\tilde{\theta}_{21})+\frac{1}{\gamma_{22}}\tilde{\theta}_{22}\operatorname{sgn}(\tilde{\theta}_{22})+\frac{1}{\gamma_{24}}\tilde{\theta}_{24}\operatorname{sgn}(\tilde{\theta}_{24})\\ &+\frac{1}{\gamma_{42}}\tilde{\theta}_{42}\operatorname{sgn}(\tilde{\theta}_{42})+\frac{1}{\gamma_{43}}\tilde{\theta}_{43}\operatorname{sgn}(\tilde{\theta}_{43})+\frac{1}{\gamma_{44}}\tilde{\theta}_{44}\operatorname{sgn}(\tilde{\theta}_{44})\end{aligned} \quad (5)$$

In (5) $\tilde{\theta}_{ij}=\theta_{ij}-\theta_{ij}^{*}$, $i=2$, $j=1,2,4$. and for $i=4$, $j=2,3,4$. θ_{ij}^{*}'s mean the true values of adaptive laws.

The symbol 'sgn' in Lyapunov function candidate means the known sign function and we use this function to guarantee positiveness of respective terms, instead of using pure quadratic Lyapunov function. This new structure of Lyapunov like function, which is chosen in this paper, has certain advantages in comparison with quadratic structure, because in this way, we avoid division by adaptive terms in the adaptive laws, if there is a need in the design. We however, redefine sign function for our purposes, to avoid uncertainty at zero point, when taking Lyapunov derivative, [6, 7], i.e.

$$\mathrm{sgn}(x)=\begin{cases}1, if\, x\geq 0\\ -1, if\, x<0\end{cases}$$

The need to redefine this function is because for applying the Lyapunov theorem we must assure positive definiteness of (5). γ_{ij} 's., for $i=2,\ j=1,2,4$, and for $i=4,\ j=2,3,4$ are small positive constants.

For Lyapunov derivative of (5) along the trajectories of (1), we have

$$\begin{aligned}\dot V(t) &= x_1 \dot x_1 + \dot x_1 \ddot x_1 + x_2 \dot x_2 + \dot x_2 \ddot x_2\\ &+\dot\theta_{21}\,\mathrm{sgn}(\tilde\theta_{21})+\dot\theta_{22}\,\mathrm{sgn}(\tilde\theta_{22})+\dot\theta_{24}\,\mathrm{sgn}(\tilde\theta_{24})\\ &+\dot\theta_{43}\,\mathrm{sgn}(\tilde\theta_{43})+\dot\theta_{44}\,\mathrm{sgn}(\tilde\theta_{44})+\dot\theta_{42}\,\mathrm{sgn}(\tilde\theta_{42})\end{aligned} \tag{6}$$

Substituting from (4) into (6), adding and subtracting terms $K_1x_1^2$, $K_2\dot x_1^2$, $K_3x_2^2$, $K_4\dot x_2^2$, where K_i 's, $i=1,\ldots,4$ are positive constants, and after some algebra, we obtain for adaptive laws, (7). Simultaneously, we have used LaSalle's invariance principle [9], in the manner, as in [8], and currently using in every manuscript on adaptive control, see for example, [7, 13, 14].

$$\begin{aligned}&\dot\theta_{21}(t)=-\gamma_{21}\,\mathrm{sgn}(\tilde\theta_{21}(t))(K_1x_1^2+0.5x_1\dot x_1+(a_{21}-b_{22}\theta_{21}(t))x_1\dot x_1)\\ &\dot\theta_{22}(t)=-\gamma_{22}\,\mathrm{sgn}(\tilde\theta_{22}(t))(K_2\dot x_1^2+0.5x_1\dot x_1+b_{22}\theta_{22}(t)\dot x_1^2))\\ &\dot\theta_{24}(t)=-\gamma_{24}\,\mathrm{sgn}(\tilde\theta_{24}(t))(a_{24}-b_{22}\theta_{4}(t))\dot x_1\dot x_2\\ &\dot\theta_{43}(t)=-\gamma_{43}\,\mathrm{sgn}(\theta_{43}(t))(K_3x_2^2+0.5x_2\dot x_2+(a_{43}-b_{41}\theta_{43}(t))x_2\dot x_2)\\ &\dot\theta_{44}(t)=-\gamma_{44}\,\mathrm{sgn}(\tilde\theta_{44}(t))(K_4\dot x_2^2+0.5x_2\dot x_2+b_{41}\theta_{44}(t)\dot x_2^2))\\ &\dot\theta_{42}(t)=-\gamma_{42}\,\mathrm{sgn}(\tilde\theta_{42}(t))(a_{42}-b_{41}\theta_{42}(t))\dot x_1\dot x_2\end{aligned} \tag{7}$$

For Lyapunov derivative (6), we finally obtain

$$\dot V(t)=-K_1x_1^2-K_2\dot x_1^2-K_3x_2^2-K_4\dot x_2^2 \tag{8}$$

6. STABILITY AND CONVERGENCE OF THE PROPOSED ADAPTIVE CONTROLLER

Following the results found in Ioannou et al., [5] we can prove the stability of the proposed controller in a similar way.

i) Stability. Because from equations (5) and (6) is seen that (5) is Lyapunov function, applying the well known Lyapunov stability theorem [5, p. 110], (local stability), one can conclude that state vector $x=[x_1,\ \dot x_1,\ x_2,\ \dot x_2]^T$ and vector of adaptive laws $\theta=[\theta_{21},\ \theta_{22},\ \theta_{24},\ \theta_{42},\ \theta_{43},\ \theta_{44}]^T$ are bounded, i.e. $x,\ \theta\in L_\infty$. From (5) and (6) also follows that state vector and its rate are $x,\dot x\in L_2$, [5]. Applying corollary from Section 2 we are led to the conclusion that state vector $x(t)\to 0$ as $t\to\infty$.

ii) Convergence. To show convergence of the first adaptive law in (9), we can write it in the following way. The proof for the remainders of equations in (9) is similar.

$$\dot\theta_{21}(t)=f_1(t)+\tilde\theta_{21}(t)f_2(t) \tag{9}$$

where we made the substitutions

$$\begin{aligned}&f_1(t)=-\gamma_{21}\,\mathrm{sgn}(\tilde\theta_{21}(t))(K_1x_1^2+0.5x_1\dot x_1+\hat a_{21}x_1\dot x_1)\\ &f_2(t)=\gamma_{21}\,\mathrm{sgn}(\tilde\theta_{21}(t))\hat b_{22}x_1\dot x_1\end{aligned} \tag{10}$$

We rewrite (9) in the form

$$\dot\theta_{21}(t)=\theta_{21}(t)f_2(t)+(f_1(t)-\theta_{21}^{*}(t)f_2(t)) \tag{11}$$

Equation (11) is nonhomogeneous differential equation of first order, and its solution exists and is finite, because expressions $f_1(t)$ and $f_2(t)$ in (10) are bounded. This is true because state vector and adaptive laws are bounded, as was proven in (i) above.

With that, convergence of adaptive laws and the stability of whole adaptive stabilizing scheme (2) or (1), (3), and (9) is proven.

7. NUMERICAL SIMULATIONS

In numerical simulations presented in this section, we consider the model given by (2), where the parameters are related with parameters of linear model of pendulum (1), with the following expressions $a_{21}=g/\Theta$, $a_{24}(t)=-\alpha\omega_0(t)/\Theta$, $a_{42}(t)=\omega_0(t)/\Theta$, $a_{43}=g/\Theta$, $b_{22}=1/\Theta$, $b_{41}=-1/\Theta$. For coefficient Θ we have $\Theta=J/ml$, [1]. In our case inertial moment is $J=0.0039$, mass of the pendulum is $m=1.5\ kg$, and the length of the pendulum is $l=0.25\ m$, $\Theta=0.0103$, $\alpha=0.1$. For the spin torque we took the value $\omega_0=10.0$ r/s. For the parameters of the model (2) we have the following values $a_{21}=954.4865$, $a_{24}(t)=-97.2973$, $a_{42}(t)=972.973$, $a_{43}=954.4865$, $b_{22}=97.2973$, $b_{41}=-97.2973$.

To avoid complicity of using the proposed adaptive control law we drop last expressions in each of two expressions in (3), and use the control law below as in the case, where it is no spin torque acting on the pendulum. (We included spin momentum in the model only for theoretical purposes and complete investigations of that problem will be made in further works.). But, because of robustness properties of the proposed adaptive control scheme, we add in the model as initial spin torque $\omega_0=10.0$ r/s.

Adaptive laws adjusted during the simulations are

$$\begin{aligned}&\dot\theta_{21}(t)=30.0\,\mathrm{sgn}(\tilde\theta_{21}(t))\times\\ &\qquad\times(-x_1^2-0.5x_1\dot x_1+727.5x_1\dot x_1-74.1\theta_{21}(t)x_1\dot x_1)\\ &\dot\theta_{22}(t)=0.7\,\mathrm{sgn}(\tilde\theta_{22}(t))(-\dot x_1^2-0.5x_1\dot x_1-74.1\theta_{22}(t)\dot x_1^2)\end{aligned}$$

$$\begin{aligned}&\dot\theta_{43}(t)=70.0\,\mathrm{sgn}(\tilde\theta_{43}(t))\times\\ &\qquad\times(-x_2^2-0.5x_2\dot x_2+727.5x_2\dot x_2-74.1\theta_{43}(t)x_2\dot x_2)\\ &\dot\theta_{44}(t)=0.3\,\mathrm{sgn}(\tilde\theta_{44}(t))(-\dot x_2^2-0.5x_2\dot x_2+74.1\theta_{44}(t)\dot x_2^2)\end{aligned}$$

Comparing the used adaptive laws with (7) and given real parameters of pendulum on the beginning of this section, it is clear that we use very rough *a priory* estimations of the parameters, i.e. $\hat a_{21}=727.5$, $\hat a_{43}=727.5$, $\hat b_{22}=74.1$, $\hat b_{41}=-74.1$.

Simulation results are represented in figures 1 to 3.

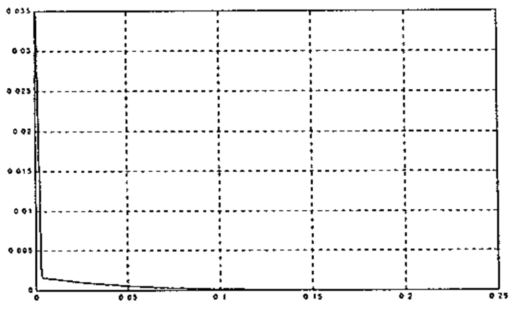

Fig. 1 – First Angle Position ψ .

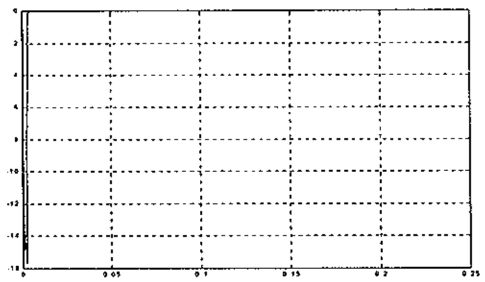

Fig. 2 – First Angle Velocity $\dot{\psi}$.

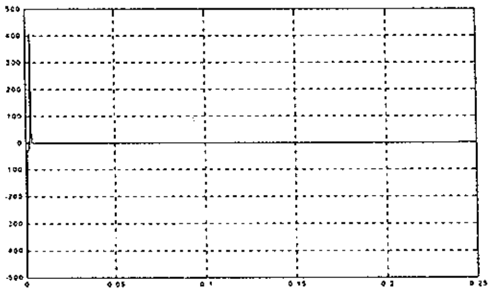

Fig. 3 – First Control Torque $u_1(t)$.

These three figures show respectively stabilization of the first angle ψ , figure 1, its velocity $\dot{\psi}$, figure 2, and the last of these three figures shows the evalution of the first control torque $u_1(t)$.

Results from simulations for another angle are similar and are not presented because of space limitations.

In conclusion of this section we should note that our adaptive scheme works satisfactory enough and its performance is measurable with time period of freely falling of the pendulum.

CONCLUSIONS

The adaptive controller, synthesized in this paper as indirect one does not need current estimations of plant parameters and it works with comparatively rough *a priory* estimation of parameters. This confirms good robust properties of the proposed adaptive scheme. On the other hand, this means that independently of the fact that the controller is designed as indirect, it behaves really as direct one. Simulations confirm simultaneously, when spin torque is small enough, interconnecting transfer functions, which in this case are unstable, can be treated as modeling errors and n controllers may be designed for every output, see please [5, p.735].

One of disadvantages of the proposed adaptive controller is that it needs on line measurements or estimation of full state vector, i.e. in this case, the values of position and velocity. Further developments can be realized with use of velocity estimator or , searching for stabilizing controller only on the basis of position measurements, i.e. design of output semiglobally stabilizing controller [10].

The main advantages of this design, in which is proposed new kind of Lyapunov function, are: (i) we are not taken into account the relative degree of transfer functions in main channels, and also of coupling transfer functions, (ii) we do not apply Lyapunov equation, which can not be solved in this case, because state matrices are not Hurwitz. This implies on the next step the results like Meyer-Kalman-Yakubovich Lemma are not applicable, further in the design, [5], (iii), we do not use reference model, but as is seen from adaptive laws (7), see also please section 7, we have the possibility to change the rate of convergence.

The big advantage of the proposed approach is that it could be applied on unstable plants, as was demonstrated for this case.

REFERENCES

1. Reithmeier, E., S. P. Stanchev, Adaptive controller design for a derived model of an inverted pendulum, taking into account the spin torque acting eventually on it. Theory and simulations, (submitted for publication)
2. Morse, A. S., A three-dimensional universal controller for the adaptive stabilization of any strictly proper minimum-phase system with relative degree not exceeding two, IEEE Trans. Automat. Contr., Vol. 30, 1985, pp. 1188-1191
3. Miyasato, Y., A design method of universal adaptive stabilizer, IEEE Trans. Automat. Contr., Vol. 45, 2000, pp. 2368-2373
4. Morse, A. S., A 4(n+1)-dimensional model reference adaptive stabilizer for any relative degree one or two, minimum phase system of dimension n or less, Automatica, Vol. 23, 1987, pp. 123-125
5. Ioannou, P. A., J. Sun, *Robust Adaptive Control*, Prentice-Hall, 1996
6. Coddington, E. A., N. Levinson, *Theory of Ordinary Differential Equations*, McGraw-Hill, N.Y., 1955
7. Narendra, K. S., A. M. Annaswamy, *Stable Adaptive Systems*, Prentice-Hall, 1989
8. Kaufman H, I. Bar-Kana, K. Sobel, *Direct Adaptive Control Algorithms: Theory and Applications*, Springer-Verlag, N.Y., Inc., 1994
9. LaSalle, J., Stability of nonautonomous systems, Nonlinear Analysis Theory. Methods and Applications, Vol. 1, 1994, pp. 83-88
10. Teel A., L. Praly, Tools for semiglobal stabilization by partial state and output feedback, SIAM J. Contr. And Opt. Vol. 33, 1995, pp. 1443-1488
11. Petersen, J. R., Nonlinear versus linear control in the direct output feedback stabilization of linear systems, IEEE Trans. Automat. Contr., Vol. 30, 1985, pp. 799-802
12. Lozano, R., I. Fantoni, D. J. Block, Stabilization of the inverted pendulum around its homoclinic orbit, Systems & Control Letters, 40, 200, pp 197-204
13. Sastry, S., M. Bodson, *Adaptive Control. Stability, Convergence, and Robustness*, Prentice-Hall, 1989
14. Goodwin, G. C., K. S. Sin, *Adaptive Filtering, Prediction, and Control*, Prentice-Hall, 1984
15. Tsachouridis, V. A., G. A. Medrano-Cerda, Discrete-time H_∞ control of a triple inverted pendulum with single control input, IEE Proc. D, Control Theory and Applications, Vol. 146, No. 6, pp. 567-577, Nov. 1999