Optical Measurements on Vibrations of Rotating, Radial Symmetric Objects

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Abstract

Vibration measurements of rotating Objects are of high interest in many industrial applications. Within this paper, an optical measurement system for non-contact measurements of "out-of-plane" vibrations of rotating objects is presented. The measurement system consists of a Scanning Laser Doppler Vibrometer (SLDV) and Optical Image Derotator. Due to the combination of the SLDV and the Derotator, the measurement system is capable of measuring vibrations in a reference coordinate system that is fixed to the rotating object. Measurement data, which is obtained during vibration measurements of a rotating saw blade is presented. The vibration measurements were carried out in a reference coordinate system that was fixed to rotating blade as well as to the lab. To explain the difference in the measurement data which is obtained in the two different reference coordinate systems a mathematical model is introduced.

Keywords:
Vibration measurement, optical image Derotator, laser-doppler-vibrometer, mode-splitting

1 INTRODUCTION

Vibration measurements of rotating objects have gained increasing interest during the last century. Especially vibration of radial symmetric objects are of great interest in many industrial applications, e. g. rotating hard disks or rotating saws for wood or stone cutting.

The Investigation of Eigenfrequencies and Eigenmodes of vibration of an object is very important. For example, the excitation of a saw blade with an Eigenfrequency of the blade can lead to the destruction of the blade.

The interest for determining vibrations of radial symmetric objects has led to numerous theoretical models some of which have already been published at the beginning of the last century. In 1922 Southwell [1] presented one of the first analytical models, which describes the vibration of a radial symmetric disk clamped in the middle. Even the early mathematical models show that a frequency analysis of the vibration a rotating object strongly depends on the reference coordinate system in which the measurement is performed in. A measurement system that rotates with the rotating object measures different Eigenfrequencies than a measurement system that is fixed to the lab.

Hence, it is desirable to be able to measure vibrations in a coordinate system that is fixed to a rotating object. There exist different approaches in literature for this purpose. E. g., Perez-Lopez *et al.* [2] used an optical image Derotator in combination with double exposure holography. The advantage of this method is that the vibration of the object can be measured at once. A drawback is the great sensitivity to vibration noise.

Pahlitzsch and Rowinski [3] used inductive probes that were rotated with the object. However, the method is very extensive since a great number of single probes have to be used in order to measure the modes of vibration.

Within this paper, a method, which is based on an Optical Image Derotator and a Scanning Laser Doppler Vibrometer, is presented.

2 EXPERIMENTAL SETUP AND METHODOLOGY

The experimental setup is shown in figure 1. The optical image Derotator consists of a Dove prism. The prism was placed in a hollow shaft motor. A control unit was used to rotate the prism at exactly half the angular frequency ω of the rotating sample. The Derotator was placed in front of the object such that the optical axis of the prism matched the axis of rotation of the sample. A circular saw blade was used as sample.

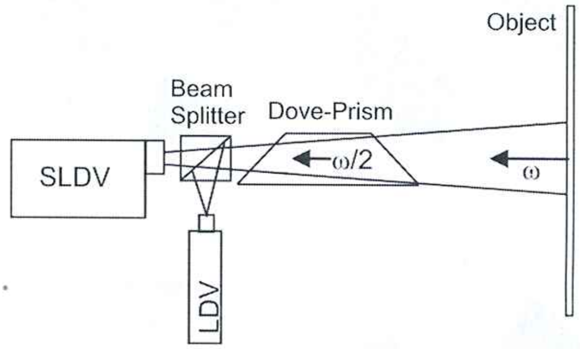

Figure 1: Experimental setup, © IMR

The vibration measurement unit consists of two different Laser-Doppler-Vibrometers. The first Vibrometer was used in combination with a mirror unit. The purpose of the mirror unit was to direct the laser beam of the Vibrometer on different measurement points on the sample. The Eigenmodes of the sample can be determined by measuring the vibration on discrete points. We will refer to the first Vibrometer as "Scanning Laser Doppler Vibrometer" (SLDV).

For reconstruction of the Eigenmodes of vibration a second Vibrometer must be used. It is necessary to use a second Vibrometer because the first Vibrometer looses the phase information of the vibration at different measurement points during the scanning process. The phase information can be obtained by measuring the vibration at a fixed point on the sample with the reference Vibrometer. The Laser beam of the second Vibrometer was coupled into the prism using a beam splitter.

The Vibration measurements were carried out using a circular saw with the diameter of 1 m and flange diameter of 0,335 m. 64 discrete measurement point were used for the reconstruction of the Eigenmodes of vibration. The vibration was measured three times at each measurement point. The Spectrum was calculated by averaging the absolute value of the Fast Fourier Transform of the three measured time signals.

The measurements were carried out at sample rotation frequencies ν_D = 0 Hz, 9.7 Hz, 13.1 Hz and 15.9 Hz. The Eigenfrequencies and Eigenmodes of vibration for each rotation frequency were measured with and without

Derotator in order to determine the difference between measurements in a sample fixed and a lab fixed reference coordinated system. The vibration of the saw blade was excited using a shaker.

3 EXPERIMENTAL RESULTS

Some Eigenmodes of the saw blade are shown in figure 2 for a rotation frequency $\nu_D = 30$ Hz. The corresponding Eigenfrequencies are denoted with ν_E. Figure 2a shows Eigenmodes, which were measured using the Derotator (reference coordinate system fixed to the sample), and Figure 2b shows Eigenmodes, which were measured without Derotator (reference coordinate system fixed to the lab). In comparison, the shapes of the Eigenmodes are identical in both cases. However, the corresponding Eigenfrequencies differ and in the case of a lab fixed reference coordinate system, there are two Eigenfrequencies showing the same shape of the mode of vibration.

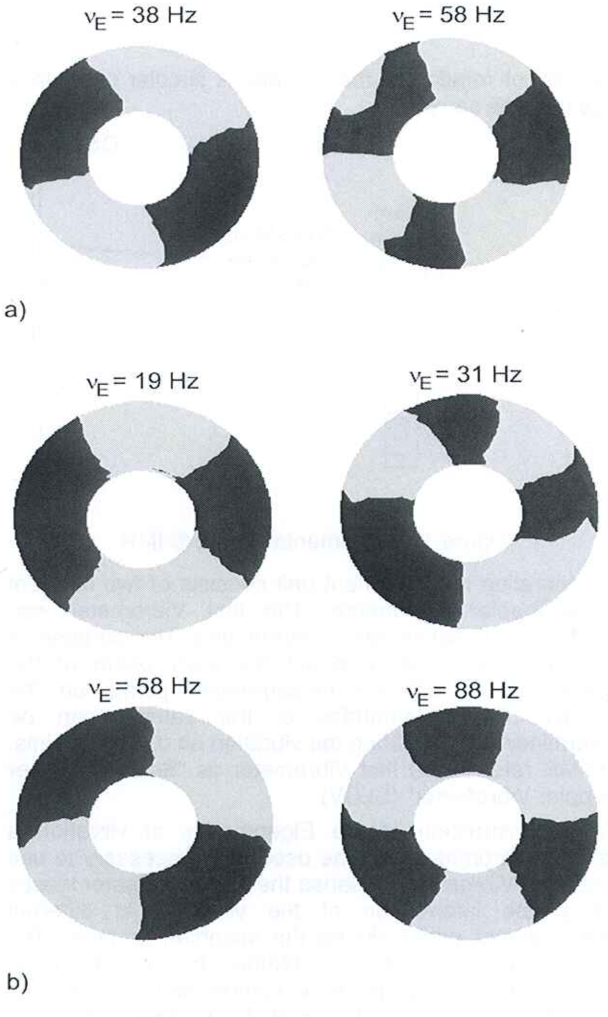

Figure 2: Eigenmodes and Eigenfrequencies E of vibration for a circular saw blade (rotation frequency (D = 30 Hz): a) measurements with Derotator, b) measurements without Derotator, © IMR

The measured Eigenfrequencies of the saw blade are shown in figure 3 as a function of the rotation frequency (ν_D = 0 Hz, 30 Hz, 40 Hz and 50 Hz). The solid lines correspond to Eigenfrequencies showing the same shape of the mode of vibration, which are measured with Derotator. The dashed lines correspond to measurements without Derotator, respectively.

The results of the measurements, which were carried out with Derotator show an increase of the Eigenfrequencies with an increasing frequency of rotation which is due to an increase of the centrifugal force.

The results of measurements, which were carried out without Derotator show that a rotation leads to two Eigenfrequencies showing the same shape of Eigenmodes. Hence, compared to the Eigenfrequencies, which are measured with Derotator, the Eigenfrequencies measured without Derotator split up into two Eigenfrequencies leading to two branches in figure 3 (dashed lines).

4 DISCUSSION

In order to explain the difference between the Eigenfrequencies measured in a reference coordinate system, which is fixed to the sample and which is fixed to the lab, a rotating mode of vibration having M nodal lines of vibration at an Eigenfrequency ν_E is considered. A measurement at a measurement point that rotates with the blade shows an out-of-plane displacement

$$S_R(t) = A_0 \sin(2\pi\nu_E t)\,, \tag{1}$$

where t and A_0 denote the time and the amplitude, respectively. When measuring in a lab fixed coordinate system, the measured vibration is modulated by the nodal lines passing the measurement point. Hence, the measured displacement is given by

$$\begin{aligned} S_L(t) &= A_0 \cos\left(2\pi\nu_E t\right)\cdot\cos\left(2\pi\cdot M\cdot\nu_D t\right) \\ &= \frac{A_0}{2}\cdot\cos\left(2\pi(\nu_E + M\cdot\nu_D)\cdot t\right) \\ &\quad + \frac{A_0}{2}\cdot\cos\left(2\pi(\nu_E - M\cdot\nu_D)\cdot t\right). \end{aligned} \tag{2}$$

The first cosine in equation 2 represents the vibration and the second cosine represents the modulation due to the passing nodal lines. Since a Fourier transform consists of a superposition of harmonic functions, a frequency analysis of equation 2 will result in two Eigenfrequencies, namely $\nu_E - M\nu_D$ and $\nu_E + M\nu_D$.

Hence, if one measures the vibration of a rotating circular symmetric object in a coordinate system, which rotates with the object, the modes and Eigenfrequencies are measured correctly. If one measures the vibration in a coordinate system, which is fixed to the lab, for each Eigenfrequency two frequencies are measured. The frequency difference of the two frequencies depends on the rotation frequency and the number of nodal lines M. This effect is known as "mode-splitting". The results agree with those of Pahlitzsch and Rowinski [3].

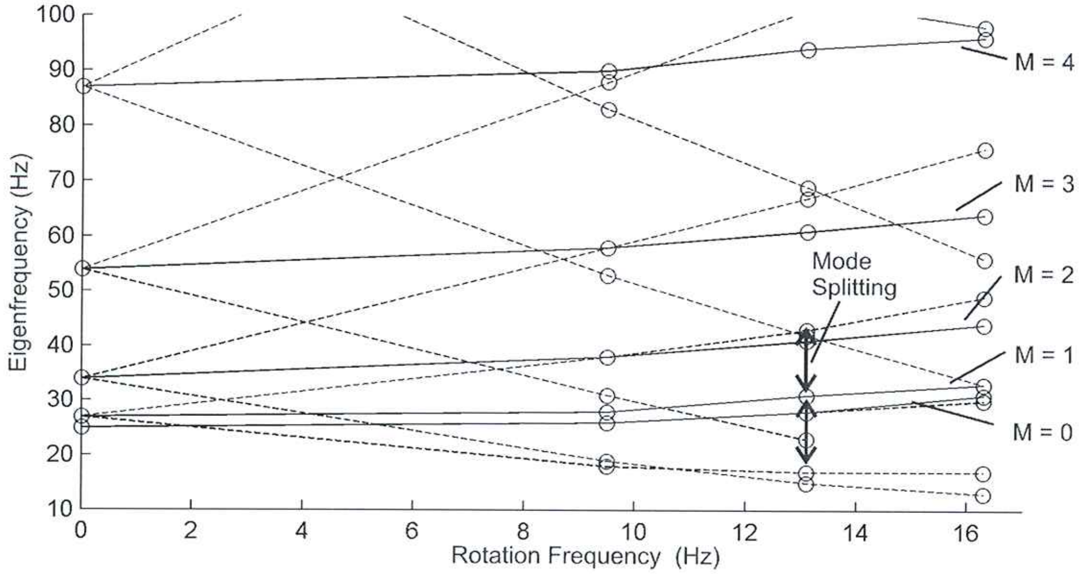

Figure 3: Campbell diagram: Eigenfrequencies measured with Derotator (solid lines) and without Derotator (dashed lines), © IMR

5 SUMMARY

An optical measurement system, which consists of an image Derotator and a Scanning-Laser-Doppler-Vibrometer (SLDV), is presented. Using the SLDV Eigenfrequencies and shapes of the modes of vibration can be determined. The Laser of the SLDV is guided through the optical Derotator such that one can measure vibrations of a rotating object in a coordinate system, which rotates with the object.

Results of vibration measurements of a circular saw blade, which were carried out with and without Derotator, are presented. The experimental results show that the measured Eigenfrequencies strongly depend on the coordinate system in which the measurement is performed in. A measurement with a SLDV leads to different result compared to measurements were the laser beam of the SLDV is directed through the Derotator. In the latter case, the laser beam will stick to a measurement point, which is fixed to the rotating object.

In order to explain the difference of both measurements a simple mathematical model of the measurement principal is introduced.

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