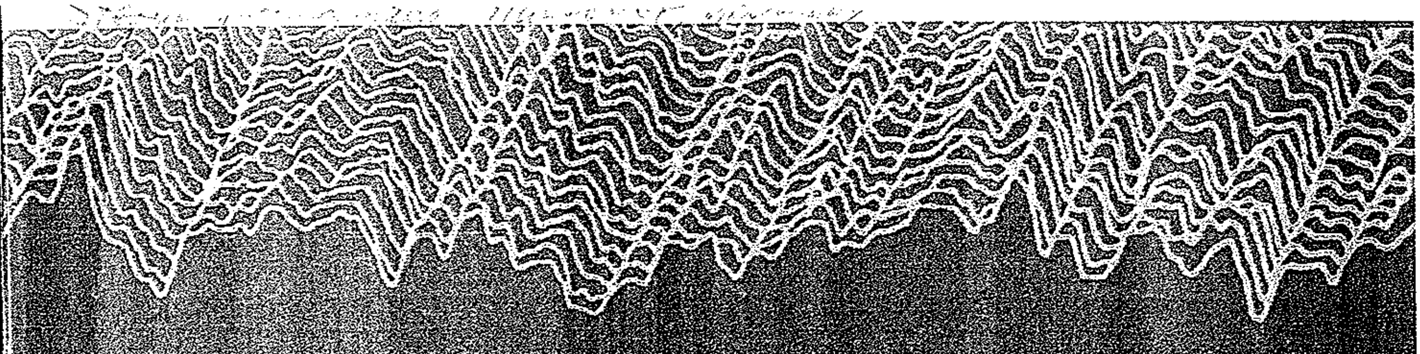

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Accessing roughness in three-dimensions using Gaussian regression filtering

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1. Introduction

During the last years 3-D description and analysis of surface geometry has gained more and more importance, since functional behaviour of engineered surfaces is, aside from depth of roughness, influenced by geometrical form of micro structure elements and their spatial distribution. Additionally form and spread of those elements is determined by modern manufacturing processes. EBT(Electrical Beam Texturing)-deep-drawing sheet steel and cylinder liners from cast aluminium-silicon-alloys can be referred to as practical examples for functionally orientated surfaces. In order to ensure sensible analysis of the surface roughness including its structure elements, it has to be separated from longer waved surface components like form and waviness by appropriate signal processing techniques (ISO 4287 [1]). The reference surface determined by the filtering method serves as a basis for quantitative analysis of roughness and has an immediate effect on the calculation of the surface parameters. A powerful tool to gain such a reference is the 3-D regression filter developed at the Institute for Measurement and Control, University of Hanover, which has the following features [2,3,4,5]:

- There is a continuous filter concept for 2-D and 3-D surface analysis.
- The filter works without any running in and - out sections, there are no transient effects. The full measured area can be analysed.
- Robustifying allows the distortionless filtering of plateaulike surfaces with deep scores, such as plateau honed cylinder liners for example.
- The filter will work impeccably with distinctive forms too.
- It is defined in spatial as well as in frequency domain.
- Transmission characteristics are comparable to those of the Gaussian filter (ISO 11562 [6]).

After looking at the basics of regression filter techniques this article will discuss the 3-D Gaussian regression filter of zero order. The regression filter of second order should be applied to workpieces with distinct form components. Thirdly a robust algorithm suitable for functionally stratified surfaces with asymmetrical amplitude distribution is introduced.

2. Regression filtering technique

A regression filter can be described by the following minimising problem [2,4,5]:

$$\int_0^{lt_y}\int_0^{lt_x} \left(z(\xi,\eta)-w(x,y)\right)^2 \cdot \tilde{s}(x-\xi,y-\eta)\cdot d\xi\cdot d\eta \rightarrow \underset{w(x,y)}{Min},\ 0\le x\le lt_x, 0\le y\le lt_y. \quad (1)$$

This double integral over the measured surface is compound of two terms. The first term contains the error square between the measured surface $z(x,y)$ and the reference plane $w(x,y)$. The second term represents the local variant weighting function of the low-pass filter $\tilde{s}$ which results from a local invariant weighting function s and the local variant correction function $\tilde{c}$ [2]

$$\tilde{s}(x-\xi,y-\eta)=s(x-\xi,y-\eta)\cdot\tilde{c}(x-\xi,y-\eta). \quad (2)$$

This local variant weighting function conforms for every position (x,y) the volume condition

$$\int_0^{lt_y}\int_0^{lt_x} \tilde{s}(x-\xi,y-\eta)\cdot d\xi\cdot d\eta = 1. \tag{3}$$

In order to take the approximation of form by the filter into consideration we suppose, that the form $f(x,y)$ at a point (x,y) in the measuring plane can be indicated by a Taylor's Series of n-th order

$$f(\xi,\eta) = \sum_{j=0}^{n}\frac{1}{j!}\sum_{h=0}^{j}\binom{j}{h}\cdot\left.\frac{\partial^j f(\xi,\eta)}{\partial\xi^j\cdot\partial\eta^{j-h}}\right|_{\xi=x,\eta=y}\cdot(\xi-x)^h\cdot(\eta-y)^{j-h}. \tag{4}$$

If $f(\xi,\eta)$ is inserted into the minimising problem from equation 1 and the filter plane $w(x,y)$ shall approximate the form correctly, a multidimensional moment condition results additionally aside the volume condition [2]

$$\int_0^{lt_y}\int_0^{lt_x}(x-\xi)^h\cdot(y-\eta)^{j-h}\cdot\tilde{s}(x-\xi,y-\eta)\cdot d\xi\cdot d\eta = 0,\quad j=1,2,\ldots,n,\quad h=0,1,\ldots,j. \tag{5}$$

Therefore a correction function $\tilde{c}$ for the regression filter is required, so that the modified weighting function $\tilde{s}$ conforms to the condition of both, the volume and the moment. Equation (1) leads to the local variant convolution integral as calculation method for the filter surface.

$$w(x,y) = \int_0^{lt_y}\int_0^{lt_x} z(\xi,\eta)\cdot\tilde{s}(x-\xi,y-\eta)\cdot d\xi\cdot d\eta \tag{6}$$

The Gaussian probability function will now be utilised as a weighting function of the low pass regression filter.

2.1. 3-D Gaussian regression filter of 0 order

The local invariant weighting function of the 3-D Gaussian filter results from multiplication of two one-dimensional Gaussian probability functions. According to [2,5,7] applies

$$\begin{aligned} s(x-\xi,y-\eta) &= \frac{1}{2\pi\cdot\lambda co_x\cdot\lambda co_y\cdot\Lambda_{LP}^2}\cdot exp\left(-\frac{(x-\xi)^2}{2\cdot(\lambda co_x\cdot\Lambda_{LP})^2}-\frac{(y-\eta)^2}{2\cdot(\lambda co_y\cdot\Lambda_{LP})^2}\right) \\ &= s(x-\xi)\cdot s(y-\eta) \end{aligned} \tag{7}$$

The filter is described by the cut-off wavelengths λco_x and λco_y in x- and y-direction. Since only the volume condition has to be met, the modified weighting function $\tilde{s}$ results in [2,5]

$$\tilde{s}(x-\xi,y-\eta) = s(x-\xi,y-\eta)\cdot\tilde{c}(x-\xi,y-\eta) = \frac{s(x-\xi,y-\eta)}{\int_0^{lt_y}\int_0^{lt_x} s(x-\xi,y-\eta)\cdot d\xi\cdot d\eta}. \tag{8}$$

Figure 1 shows the shape of weighting function $\tilde{s}$ for different positions on the measured area. Regarding position 1 midst the measured areas the weighting function takes on the rotatory form of a Gaussian Bell Curve. It is only in the marginal areas (position 2 and 3) that the local dependence of the weighting function becomes obvious. Here the bell curve is cut off asymmetrically and afterwards scaled in such a way that the enclosed volume always equals 1. That allows the analysis of the margins with no loss of data.

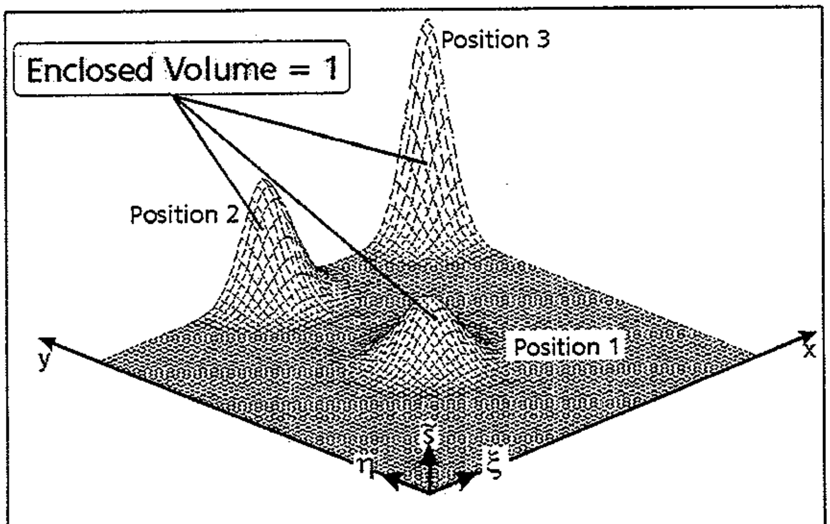

Figure 1: Shape of the weighting function of the Gaussian regression filter

Now the transmission characteristic of the filter for $0.5 \cdot \lambda co_x \leq x \leq lt_x - 0.5 \cdot \lambda co_x$ and $0.5 \cdot \lambda co_y \leq y \leq lt_y - 0.5 \cdot \lambda co_x$ shall be analysed. Here applies $\tilde{c}(x-\xi, y-\eta) \cong 1$. Accordingly from the Fourier transformation of equation (8) the transfer function comes out to [2,5,7]:

$$S(\lambda_x, \lambda_y) = exp\left(-2\pi^2 \cdot \Lambda_{LP}^2 \cdot \left(\lambda co_x^{\ 2} / \lambda_x^2 + \lambda co_y^{\ 2} / \lambda_y^2\right)\right) \qquad (9)$$

The damping of the filter for a wave front with the wavelength $\lambda_r = \lambda_x \cdot cos(\varphi) = \lambda_y \cdot sin(\varphi)$ and the angle φ therefore depends significantly on the cut-off wavelengths $\lambda co_x, \lambda co_y$. The transmission characteristic of this filter is identical to the standardised filter of ISO 11562 [1,2] if for the cut-off wavelengths $\lambda co_x = \lambda co_y = \lambda co_r$ applies.

$$S(\lambda_r) = exp\left(-2\pi^2 \cdot \Lambda_{LP}^2 \cdot \lambda co_r^{\ 2} / \lambda_r^2\right) \qquad (10)$$

For determination of the free filter parameter Λ_{LP} the amplitude of a wave front with wavelength $\lambda_r = \lambda co_r$ shall be dampened by 50%. For Λ_{LP} it comes to the value $\Lambda_{LP} = \pi^{-1} \cdot \sqrt{ln 2/2}$.

In order to demonstrate the utilisation of the filter it is more impressive to look at a profile plot instead of a 3-D presentation of the surface. Figure 2 shows a selected profile plot of a ground surface at which the regression filter of zero order approximates the waviness very well. Therefore this filter plane serves as an appropriate reference for the calculation of roughness parameters.

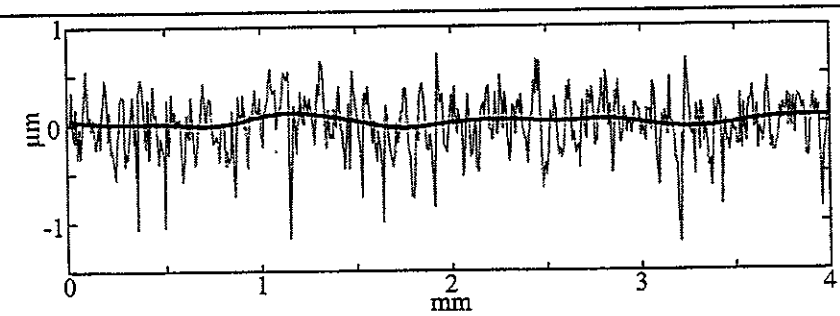

Figure 2: Applying the filter (0 order) to a ground surface (Profile extraction)

2.2. 3-D Gaussian regression filter of 2nd order (3D form filter)

The necessity to approximate higher orders of form shows Figure 3. The measured sphere (Figure 3a) has large form components. For a better visualisation Figure 3b illustrates a profile only. Especially in the marginal areas the filter of zero order (broken line) runs out of the profile, while the filter of 2nd order (solid line) delivers a good approximation. This filter shall be explained more closely now.

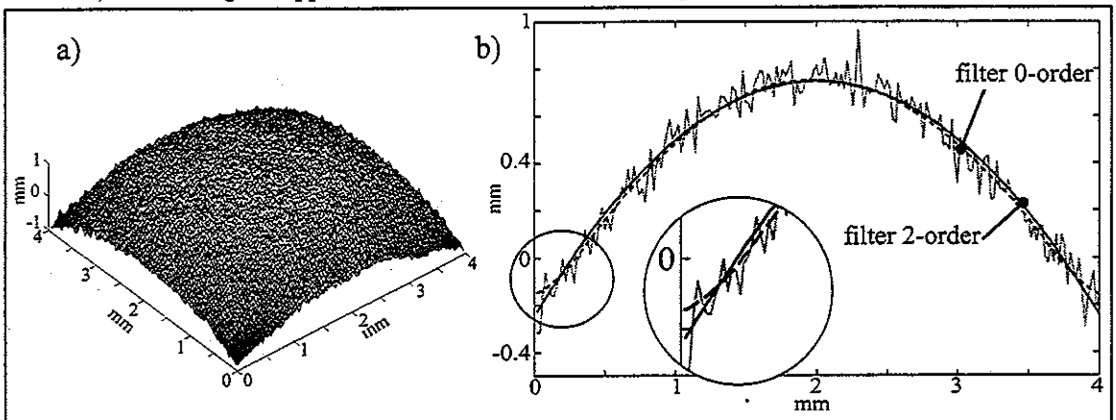

Figure 3: Form filtering: a) spherical surface example b) profile extraction

According to equation (3) and (5) six secondary conditions have to be met for implementation of the 3-D form filter if polynomials up to order $n = 2$ are to be approximated. The minimising problem can now be formulated using six parameters $\alpha_{h,j-h}$ of a polynomial surface of second order:

$$\int_0^{lt_y}\int_0^{lt_x}\left(z(\xi,\eta)-\sum_{j=0}^{2}\sum_{h=0}^{j}\alpha_{h,j-h}(x,y)\cdot(x-\xi)^h\cdot(y-\eta)^{j-h}\right)^2\cdot\tilde{s}(x-\xi,y-\eta)\cdot d\xi\cdot d\eta\rightarrow \underset{w(x,y)}{Min} \qquad (11)$$

For the determination of a function value $w(x,y)$ of the filter surface, initially the polynomial surface of 2nd order is being fitted into the surface by minimising the double integral over the deviation squares. The deviation squares are additionally weighted by the low pass weighting function $\tilde{s}(x-\xi,y-\eta)$. The function value $w(x,y)$ equals the value of the inserted surface at $\xi = x$ and $\eta = y$, according to which $w(x,y) = \alpha_{0,0}(x,y)$ applies. Due to the mixed terms $(x-\xi)\cdot(y-\eta)$ of this arrangement the calculation of the six parameters $\alpha_{h,j-h}$ leads to a complex system of equations.

Simplistically looked at a measuring surface with an infinite extension, which means both integrals in equation (11) run from -∞ to +∞, the weighting function of the 3-D form filter can be indicated with:

$$\tilde{s}_\infty(x-\xi,y-\eta)=s(x-\xi,y-\eta)\cdot\underbrace{\left(2-\frac{(x-\xi)^2}{2\cdot(\lambda cox\cdot\Lambda_{LP})^2}-\frac{(y-\eta)^2}{2\cdot(\lambda cox\cdot\Lambda_{LP})^2}\right)}_{\tilde{c}(x-\xi,y-\eta)} \qquad (12)$$

The index ∞ identifies here the operational mode of an infinitely extended measured area. The weighting function of the form filter consists of the Gaussian bell curve and the given correcting function $\tilde{c}$. For the transfer function results

$$S(\lambda_x,\lambda_y)=exp\left(-2\pi^2\cdot\Lambda_{LP}^2\cdot\left(\frac{\lambda {co_x}^2}{\lambda_x^2}+\frac{\lambda {co_y}^2}{\lambda_y^2}\right)\right)\cdot\left(1+2\pi^2\cdot\Lambda_{LP}^2\cdot\left(\frac{\lambda {co_x}^2}{\lambda_x^2}+\frac{\lambda {co_y}^2}{\lambda_y^2}\right)\right) \qquad (13)$$

out of equation (12). The damping of the filter depends here on the direction of a wave front too, as far as different values for the cut-off wavelengths are given. If $\lambda_r = \lambda_x \cdot cos(\varphi) = \lambda_y \cdot sin(\varphi)$ and $\lambda co_x = \lambda co_y = \lambda co_r$ apply, the transfer function is given by

$$S(\lambda_r) = exp\left(-2\pi^2 \cdot \Lambda_{LP}^2 \cdot \lambda co_r^2 / \lambda_r^2\right) \cdot \left(1 + 2\pi^2 \cdot \Lambda_{LP}^2 \cdot \lambda co_r^2 / \lambda_r^2\right). \tag{14}$$

For dampening of the amplitude by 50% at a wave front with a wavelength $\lambda_r = \lambda co_r$ the free filter parameter Λ_{LP} is to be fixed at $\Lambda_{LP} = 0.2916$. The transmissions characteristic of the filter plane can be demonstrated by the graph to the right of Figure 4. The black line of the form filter is steeper than the one of the regression filter of zero order marked grey in the figure. To the left, the weighting functions of both filters are given. Again the black line indicates the form filter while the grey line indicates the Gaussian filter. Distinguishing marks of the form filter are the negative weights and its larger width.
If this form filter is applied to the sphere as seen in Figure 3a) we can recognise by the filter plane in Figure 3b) the good approximation of the spherical form.

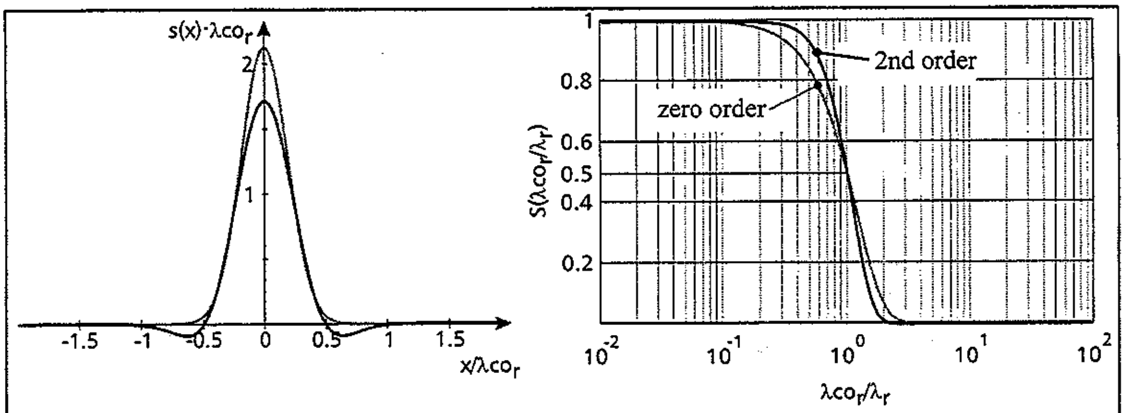

Figure 4: Weighting functions and transmissions characteristic of the regression filter of zero order (Gaussian filter)and the regression filter of second order (form filter)

3. Robust Surface Filter

With the regression filter, as so far described, form and waviness of a surface can be approximated. It is necessary though, to execute this filter robust when it comes to profile peaks and scores, just as the section of a plateau honed surface illustrates in Figure 5. The broken line represents the filter plane of the regression filter , which is being strongly influenced by the hone scores.

From the field of statistics an algorithm can be taken that modifies the regression filter to a robust filter [9,10]. For that the approach of the regression filter (equation (1)) is now generalised by a so called ρ-function. Δz is the difference of the surface ordinates $z(x,y)$ and the filter plane $w(x,y)$.

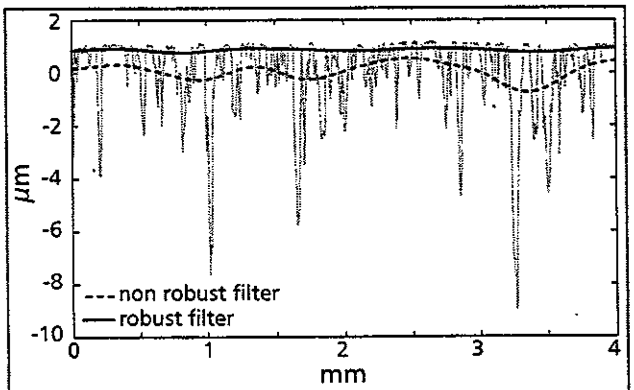

Figure 5: Profile extraction of a plateau honed surface with a non robust an robust filter line

$$\int_0^{lt_y}\int_0^{lt_x} \rho\underbrace{\left(z(\xi,\eta)-w(x,y)\right)}_{\Delta z}\cdot\tilde{s}(x-\xi,y-\eta)\cdot d\xi\cdot d\eta \rightarrow \underset{w(x,y)}{Min} \qquad (15)$$

$\rho(\Delta z)=\Delta z^2$ will result in the non robust Gaussian regression filter, as already discussed. Using the Beaton-Function as ρ-function though, a so called "hard redescendent" robust filter can be derived. Figure 6 shows the different courses of the Least-Square and the Beaton-Function. Looking at the least square function, the quadratic growing influence of Δz attracts the attention. That means, great deviations Δz as for example distinct scores enter the minimising problem with great impact. This is the reason for the misbehaviour of the filter in the plateau honed example (Figure 5).

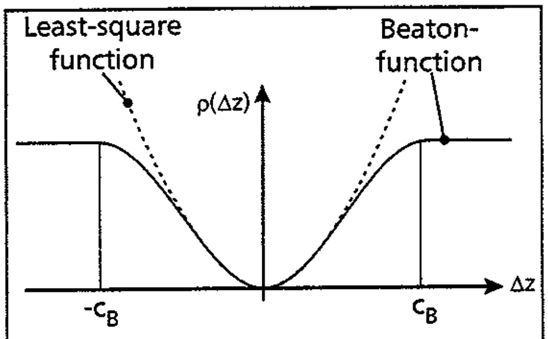

Figure 6: Least-Square-Function and Beaton-Function as ρ -functions

Opposed to that with the Beaton-Function ordinates Δz greater than a threshold value c_B enter the minimising problem only with a constant value. Profile ordinates close to the reference line are processed nearly identical by both filters.

Utilisation of the Beaton-Function leads to another minimising problem made up of the terms from equation (1) and an additional vertical weighting $\delta_i(x,y)$ for each profile ordinate:

$$\int_0^{lt_y}\int_0^{lt_x} \left(z(\xi,\eta)-w_i(x,y)\right)^2\cdot\delta_i(\xi,\eta)\cdot\tilde{s}(x-\xi,y-\eta)\cdot d\xi\cdot d\eta \rightarrow \underset{w_i(x,y)}{Min}\,. \qquad (16)$$

Since the unknown threshold value c_B has to be estimated by statistics, the filter surface must be calculated iteratively. c_B converges with engineered surfaces after 2-5 iterations. Estimated value for c_B is

$$c_B = 4.4\cdot Median\left(\left|z(x,y)-w_i(x,y)\right|\right). \qquad (17)$$

This relation is proved by statistics in [2]. The vertical weighting $\delta_i(x,y)$ for one iteration loop is calculated by [2,5,8]

$$\delta_{i+1}(x,y)=\left(1-\left(\frac{\Delta z}{c_B}\right)^2\right)^2 \quad for \quad \left|\frac{\Delta z}{c_B}\right|<1\,. \qquad (18)$$
$$\delta_{i+1}(x,y)=0 \qquad otherwise$$

The iterative algorithm is shown in Figure 7 as flowchart. Usually the filter mean plane of a non robust filtering can be taken for a first raw estimation of c_B. Then the vertical weights $\delta_i(x,y)$ are determined using c_B. The next iteration loop starts again with the calculation of a new reference plane. These iterations are repeated until the change of c_B between two iteration loops is smaller than the given tolerance value t.

Applying this robust filter to the plateau honed surface as seen in Figure 5 delivers a very good reference plane which is visualised by the black line. The introduced robust algorithm is suitable for the regression filter of zero as well as for the regression filter of 2nd order (robust form filter).

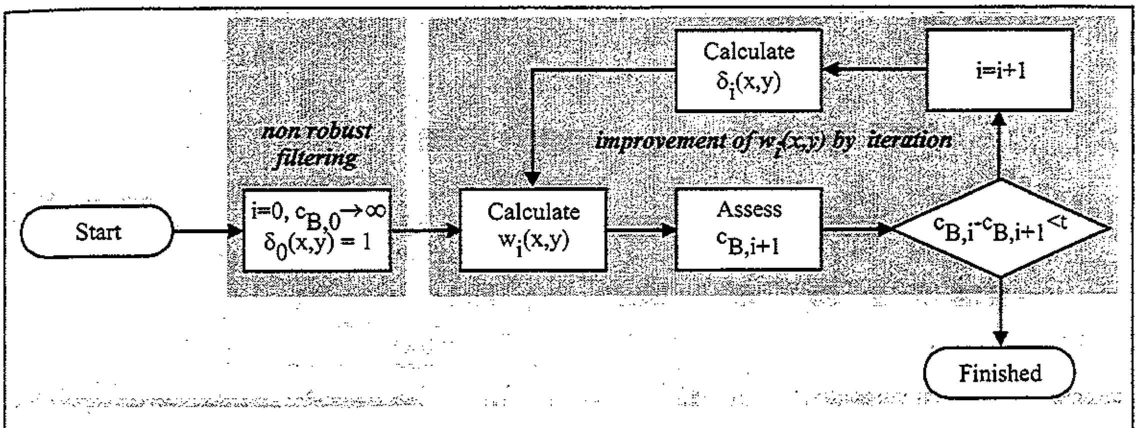

Figure 7: Flow chart for the robust algorithm

4. Summary

The robust 3-D Gaussian regression filter is an efficient tool to determine a reference plane for 3-D roughness analysis with any engineered surface. This filter can, due to the form approximation, follow even warped surfaces. Its robust algorithm was specially designed for the analysis of heavily strained contact surfaces in mechanical engineering, such as, for example, plateau honed cylinder liners. Robustifying allows the filtering of surfaces with distinct scores and peaks without distortion of the filter reference plane.

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