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**3pNS7. Design of a feedback controller for active noise control with in-ear headphones**

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This paper presents a method to design optimal feedback controllers for active noise control. It also shows the main problem of in-ear headphones, namely, interpersonal variances, and its possible solution. These variances make the design of stable feedback controllers with good noise attenuation difficult. To solve this problem we measured the secondary paths of a few test persons and divided these transfer functions into their minimum-phase and their allpass part using the complex cepstrum. In this context we show that the allpass part is almost constant. Thus, it is possible to generate an optimal controller for a given disturbance using the internal model control technique. After that the interpersonal variances will be partly compensated by a mean inverse minimum-phase transfer function. The result is a stable controller with a good noise attenuation for all test persons.

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## INTRODUCTION

Nowadays mobility is an important factor in many jobs. Therefore, there is an increased use of planes, trains and cars, and the therewith associated exposure to noise. Good acoustic insulation is often hard to realize due to the involved extra weight.

Ear protection or headphones with active noise control (ANC) may be a possible solution. Today circumaural and supra-aural ANC headphones with good attenuation are commercially available. However, their weight and the necessary headband can impair the wearing comfort. ANC in-ear headphones do not have these disadvantages and, therefore, there is a need of further research in the field of ANC.

In ANC headphones, disturbing noise is minimized by an out-of-phase anti-noise. Therefore, the noise is recorded by microphones next to each ear, and filtered by an analog or digital platform to generate the anti-noise. There are two main control strategies depending on the position of the microphones, feedforward (FF) control with an external reference microphone and feedback (FB) control with an internal error microphone [1, 2].

This paper focuses on the design of feedback controllers and the main problem in in-ear headphones, interpersonal variances, which make the design of stable controllers with high noise attenuation difficult.

Therefore, this problem is presented in the next chapter in detail, with the help of the measurements of the secondary path. In this context the measured paths will be analyzed using the complex cepstrum [3]. Then the optimal feedback controller for a given disturbance and a known secondary path will be presented. The last step completes the calculation of the FB controller by partly compensating for the interpersonal variances with a mean inverse model of the minimum-phase part. The results of simulations will show that this method is suitable to get a feedback controller with high noise attenuation for ANC applications especially with problems of in-ear headphones.

## PROBLEM STATEMENT

The presented feedback control is realized with a digital signal processor (DSP), as shown in figure 1(a). The aim of the FB control is to minimize the squared error at the error microphone:

$$e^2(n) = [d(n) - u(n)]^2, \quad (1)$$

where  $d(n)$  is the noise and  $u(n)$  is the generated anti-noise.

Figure 1(b) shows a block diagram of the feedback ANC, where  $C(z)$  is the transfer function of the digital controller and  $S(z)$  is the secondary path which includes the acoustical transfer path from the loudspeaker to the error-microphone and all electrical components of the DSP. Thus, the attenuation of the closed loop ANC system can be described by the disturbance transfer function  $F_z(z)$ :

$$F_z(z) = \frac{E(z)}{D(z)} = \frac{1}{1 + C(z) \cdot S(z)}. \quad (2)$$

We define the aim of the FB control to minimize the expected value of the squared error at the error microphone  $E[e^2(n)] = E[(d(n) - u(n))^2]$ . But (2) also shows the optimization problems:  $F_z(z)$  is non-linear in  $C(z)$ , and, as shown in figure 2,  $S(z)$  differs from person to person.

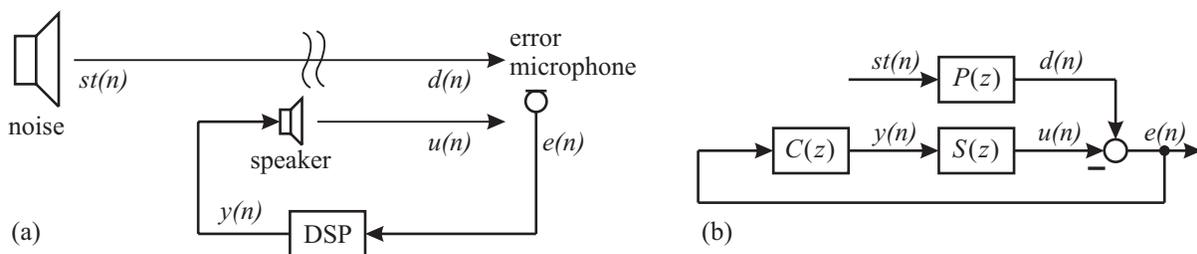


FIGURE 1. Principle (a) and block diagram (b) of feedback ANC

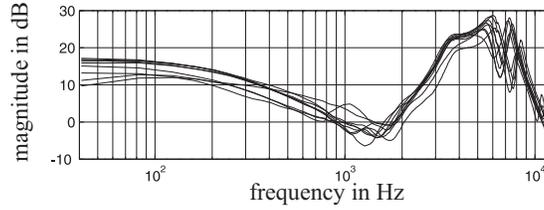


FIGURE 2. Measured secondary paths of one in-ear headphone and different test persons

## DESIGN OF AN OPTIMAL FEEDBACK CONTROLLER

In this chapter we will describe a possible solution to the problem described earlier, i.e., how to design an optimal feedback controller in terms of interpersonal variances.

First, we simplify the optimization problem. Every linear, time invariant (LTI), digital transfer function  $S(z)$  can be divided into its minimum-phase part  $S_{mp}(z)$  and its allpass  $S_{ap}(z)$ :

$$S(z) = S_{mp}(z) \cdot S_{ap}(z). \quad (3)$$

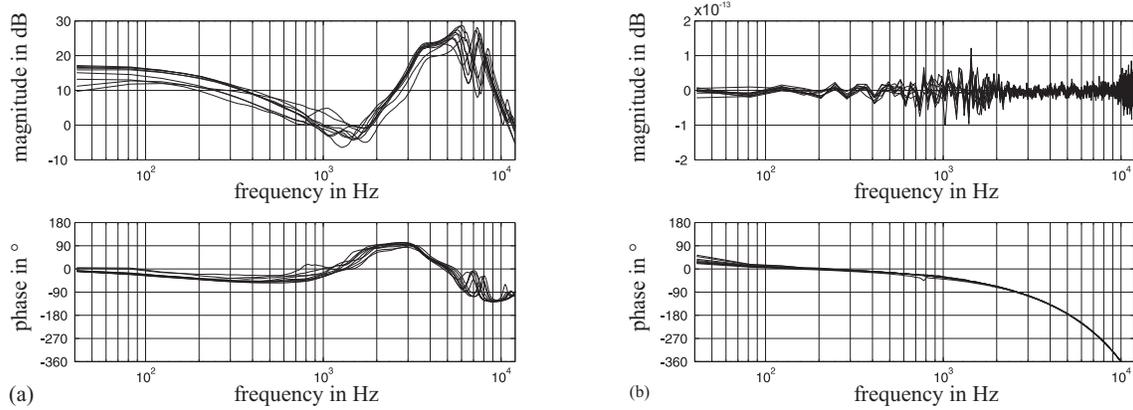


FIGURE 3. The minimum-phase part (a) and the allpass (b) of the measured secondary paths of figure 2

Using the complex cepstrum it is possible to separate the measured transfer functions of figure 2 automatically into their minimum-phase part and their allpass (figure 3). The allpass is approximately constant and the minimum-phase part is invertible for each test person. So the controller is set to:

$$C(z) = C^*(z) \cdot S_{mp}^{-1}(z), \quad (4)$$

where  $C^*(z)$  is a new configurable controller. This simplifies the disturbance transfer function (2) to:

$$F_z(z) = \frac{1}{1 + C^*(z) \cdot S_{ap}(z)}. \quad (5)$$

$F_z(z)$  is still non-linear in  $C^*(z)$ . However, it is possible for a constant allpass to use the internal model control technique [4]:

$$C^*(z) = \frac{C_{IMC}(z)}{1 - C_{IMC}(z) \cdot \hat{S}_{ap}(z)}. \quad (6)$$

Therefore, as shown in figure 4, it is possible to estimate the disturbance  $d(n)$  inside the controller with  $\hat{d}(n)$ .

If the model of the allpass  $\hat{S}(z)$  equals the real allpass  $S(z)$ , the closed loop is stable for every stable internal controller  $C_{IMC}(z)$  [5] and the disturbance transfer function (2) is simplified to a FF problem:

$$F_z(z) = 1 - C_{IMC}(z) \cdot S_{ap}(z). \quad (7)$$

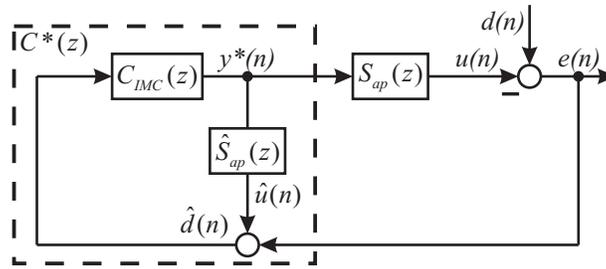


FIGURE 4. Internal model control technique

The methods to calculate the optimal controller for FF structures are well known [1, 6]. If  $C_{IMC}(z)$  is an FIR filter, the surface of the squared error  $e^2(n)$  becomes convex in every filter parameter. Thus, it is possible to use a gradient method with low computational effort, like the FXLMS algorithm, as shown in figure 5.

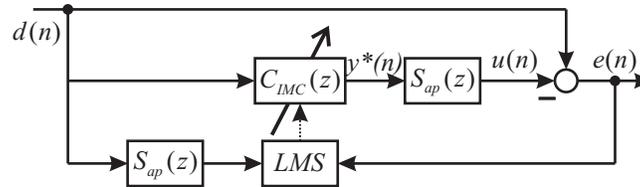


FIGURE 5. Block diagram of the filtered-reference LMS

This block diagram also shows that the optimal FB controller only depends on the allpass  $S_{ap}(z)$  and the disturbing noise  $d(n)$ . The optimal controller for one person or a constant secondary path, respectively, can be calculated as follows:

$$C(z) = S_{mp}^{-1}(z) \cdot \frac{C_{IMC}(z)}{1 - C_{IMC}(z) \cdot S_{ap}(z)}. \quad (8)$$

For the tested in-ear headphone it is possible to expand this method to all measured test persons using an inversion of a mean minimum-phase model  $\hat{S}_{mp}^{-1}(z)$ .

## RESULTS

The optimal controller was calculated with equation (6) for the disturbance  $d(n)$  (figure 6(a)) and the measured allpass  $S_{ap}(z)$  (figure 3(b)). Then we used the inversion of a mean minimum-phase model to calculate the controller  $C(z)$ . Figure 6(b) shows the simulated attenuation of this controller for all measured test persons.

The damping at the error microphone is good for all the test persons because the closed loops damp up to 400 Hz about -10 dB or even more. Furthermore, the damping effects are very similar and do not vary as much as the secondary paths (figure 2). And most importantly, the closed loop is stable for all test persons.

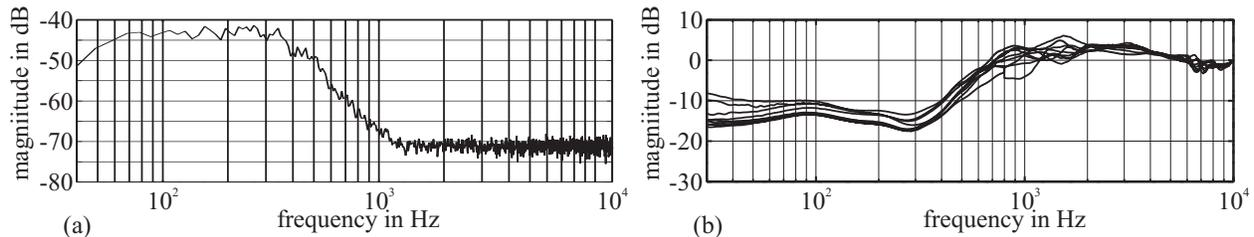


FIGURE 6. The spectrum of disturbance  $d(n)$  (a) and the attenuation  $F_Z(z)$  of different test subjects (b)

## CONCLUSION

In this paper we have shown a method to design an optimal feedback controller for ANC applications even under poor conditions, especially in case of interpersonal variances. Therefore, the secondary path was divided into a minimum-phase part and an allpass. For a given disturbance  $d(n)$ , it is possible to calculate the optimal controller for the allpass using the internal model control technique. The varying minimum-phase part can partly be compensated using a mean inverse model.

For the tested persons, this method leads to approximately similar active attenuations and stable closed loops. Instability is an important problem in this context, so it is necessary to make sure that the closed loop has a robust stability. Therefore, the amplitude and the phase margin can be analyzed. In this case the worst phase margin was about  $30^\circ$ .

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