

# Computationally Efficient and Direction Dependent Active Noise Reduction in Headsets

Jens Graf\* and Eduard Reithmeier†

*Institute for Measurement and Automatic Control  
Nienburger Strasse 17, 30167 Hanover, Germany*

(Dated: September 11, 2008)

## Abstract

In commercial circum aural ANR-headsets (Active Noise Reduction) the non-adaptive feedback control approach is mainly used due to its simplicity. Nevertheless, a disadvantage is the limited attenuation bandwidth to approximately 400Hz. If attenuation of high frequency noise is required, adaptive broadband feedforward control techniques are suggested. The implementation of such adaptive algorithms is linked to considerable computational effort which especially remains an obstacle for their realization in commercial products. In contrast to adaptive broadband feedforward control, the non-adaptive feedforward technique requires significantly less computational effort. However, when the direction of the incident noise wave and hence the optimal feedforward controller substantially varies, the attenuation performance of the non-adaptive controller significantly deteriorates.

In the framework of this paper, a noise direction dependent feedforward control approach is introduced. The controller consists of two computationally efficient non-adaptive feedforward controllers which are optimal for different representative noise directions. In order to enhance the noise reduction performance at low frequencies, an additional feedback controller is used. The actuating variable of each feedforward controller is weighted depending on the direction of the noise source and combined with the output of the additional feedback control loop. The proposed control approach significantly attenuates broadband noise while limiting the computational effort. The attenuation performance of the suggested control strategy is compared to other noise control approaches.

Keywords: Active Noise Control in Headsets, Combined Feedback and Feedforward Noise Control, Direction Dependent Feedforward Control.

## I. INTRODUCTION

Active attenuation of acoustical noise is particularly required in environments of high level noise where passive arrangements are insufficient. The most effective hearing protection can be achieved with so called ANR-Headsets (Active Noise Reduction). During the last decades different approaches have been suggested to effectively attenuate high level noise in the headset's ear-cups<sup>5</sup>. Even if very effective but also complex active systems have been developed<sup>34</sup>, contemporary ANR-Headsets still use the well known non-adaptive feedback control approach. The major reason for this is the computational complexity as well as the high costs linked to the related hardware. In general, a reasonable compromise between adequate noise attenuation and controller complexity is offered by the implementation of a non-adaptive feedback controller, which especially enables to attenuate noise in the lower frequency range. In contrast to this technique, the feedforward approach enables the attenuation of high frequency noise. However, especially in case of circum-aural ear-cups, the non-adaptive feedforward approach is sensitive to changing noise source directions. Therefore, an adaptive feedforward approach<sup>21</sup>, which is able to adapt to varying system plants, is mostly chosen. Due to the complexity, adaptive broadband feedforward ANR-systems are only suggested by researchers and commonly not implemented in commercial products. Only one commercial ANR-headset is known which uses a narrowband adaptive noise attenuation system. However, this system permits to attenuate only monofrequent sine waves rather than broadband noise.

It can be summarized that the development of broadband ANR-controllers aim at control strategies which effectively attenuate broadband noise while minimizing the system's complexity. This argumentation directly leads to non-adaptive control strategies because of their simple and easy structure. In this paper, a combined non-adaptive feedforward-feedback control approach is introduced. The feedback part of the combined controller attenuates low frequency noise whereas the feedforward controller compensates for higher frequency components. The problem of varying optimal feedforward controllers, which is directly linked to plant variations, is resolved by the implementation of a direction dependent control technique. In the following section, the acoustical components of the ANR-system are modeled and the aspect of varying system plants due to changes of the noise source direction is pointed out. Section III describes the combined control strategy consisting

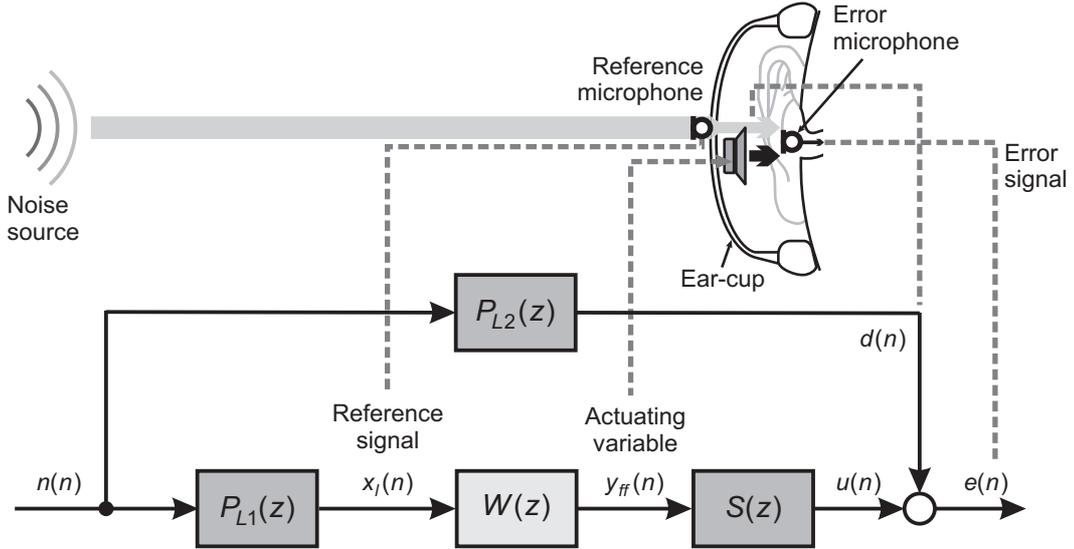


FIG. 1: Non-adaptive feedforward ANR-System. Upper part: Symbolic representation of the ANR-system. Lower part: Block diagram with the controller  $W(z)$  and the system plants  $P_{L1}(z)$ ,  $P_{L2}(z)$  and  $S(z)$  -  $S(z)$  is commonly referred to as the secondary path.

of a non-adaptive feedback control loop and a direction dependent feedforward controller. In section IV the attenuation performance of the proposed ANR-system is presented and concluding remarks are given in section V.

## II. PROBLEM STATEMENT AND SYSTEM MODELING

In the framework of broadband ANR-systems high frequency components of the disturbing noise are commonly reduced by the use of feedforward control methods. These methods commonly suffer from varying system plants which results from varying noise source directions. Therefore, recently developed broadband ANR-headsets use an adaptive feedforward control approach in order to adapt to these variations<sup>34</sup>. However, adaptive feedforward control algorithms result in complex and computationally expensive systems. In contrast, non-adaptive feedforward controllers permit to reduce the complexity and thus lead to more simple and efficient ANR-systems.

Figure 1 illustrates a non-adaptive feedforward ANR-system consisting of a circum aural ear-cup, two microphones and a compensation loudspeaker. The reference signal  $x_l(n)$  is acquired by the reference microphone and inputted to the non-adaptive feedforward con-

troller. The out-of-phase antinoise is outputted with the compensation loudspeaker and destructively interferes with the disturbing noise. While the transfer function  $P_{L1}(z)$  represents the system behavior between the noise source and the digitized reference signal, the transfer behavior from the noise source to the place of interference is symbolized by the model  $P_{L2}(z)$ . Perfect compensation of the disturbing noise at the point of interference is accomplished by minimizing the residual error  $e(n)$ :

$$\begin{aligned} e(n) &= d(n) + u(n) \stackrel{!}{=} \min \\ &\rightarrow D(z) \stackrel{!}{=} -U(z). \end{aligned} \tag{1}$$

Substituting the variables  $U(z)$  and  $D(z)$  by the system plants, the following expression is obtained:

$$N(z)P_{L2}(z) = -N(z)P_{L1}(z)W(z)S(z). \tag{2}$$

Eventually, this equation is rewritten to obtain the optimal feedforward controller:

$$W(z) = -\frac{P_{L2}(z)}{P_{L1}(z)S(z)}. \tag{3}$$

Even though the realization of the optimal controller's complete transfer function is impossible,  $W(z)$  can be designed in order to model Eq. 3 in a limited frequency range. Nevertheless, two problems occur when realizing this band limited feedforward controller. Firstly, the design of the controller assumes time invariant system plants  $P_{L1}(z)$ ,  $P_{L2}(z)$  and  $S(z)$  which is unrealistic in practical applications. Secondly, due to changing noise source directions, the optimal controller may be a non-causal transfer function. These two problems are explained in detail in the following.

#### *Problem 1: Varying System Plants*

Obviously, the system plants are time variant when the direction of the incident noise wave varies or the reflection characteristics of the surrounding changes. This is illustrated in Fig. 2. Whereas Fig 2a depicts the ANR-headset's ear-cup directly exposed to the noise source, Fig. 2b shows a completely different situation. Here, the noise source is located on the opposite side of the ear-cup. Consequently, the two different situations result in different

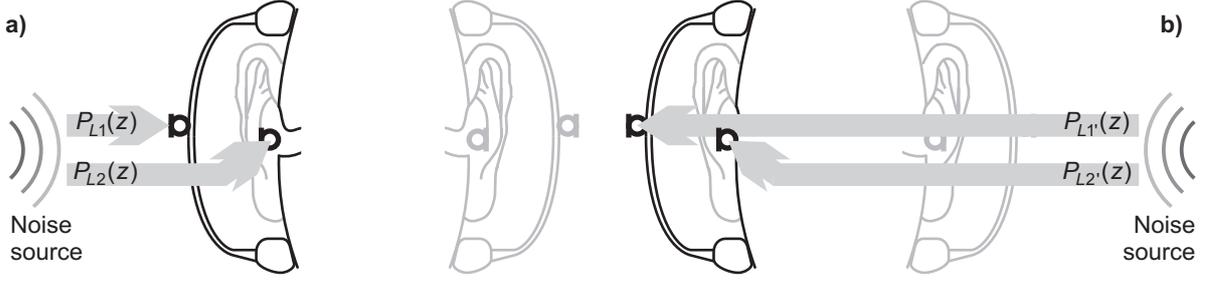


FIG. 2: Different noise situations result in different optimal controllers: a) The ear-cup is directly exposed to the noise. b) The noise source is located on the opposite side.

optimal feedforward controllers:

$$W(z) = -\frac{P_{L2}(z)}{P_{L1}(z)S(z)} \quad \text{and} \quad W'(z) = -\frac{P_{L2'}(z)}{P_{L1'}(z)S(z)}. \quad (4)$$

*Problem 2: Non-causality*

Referring to Fig. 2b, the second problem linked to non-adaptive feedforward controllers occurs when the noise signal arrives at the reference microphone delayed with respect to the error microphone. This situation can be expressed as follows:

$$\begin{aligned} P_{L1'}(z) &= P_a(z)z^{-T_1} \quad \text{and} \quad P_{L2'}(z) = P_b(z)z^{-T_2} \\ \text{with } T_2 > T_1 &\quad \rightarrow \quad T_2 = T_1 + T \end{aligned} \quad (5)$$

By definition,  $P_a(z)$  and  $P_b(z)$  are realizable transfer functions which are free of time delay.  $T_1$  and  $T_2$  denote the dead time that represent the delays between the noise source and the reference microphone as well as the delay between the reference microphone and the error microphone. Due to  $T_2 < T_1$ , the generation of an antinnoise that reaches the place of compensation at the same time as the disturbing signal is impossible. Assuming  $S(z) = 1$  for the secondary path, and inserting Eq. 5 in Eq. 4 results in a non-causal optimal feedforward controller:

$$W'(z) = -\frac{P_a(z)z^{-T_1}}{P_b(z)z^{-T_2}} = \frac{P_a(z)}{P_b(z)}z^T \rightarrow \text{non-causal!} \quad (6)$$

Due to the discussed problems above, the attenuation performance of a designed non-adaptive controller deteriorates gradually when the incident noise wave changes the direction.

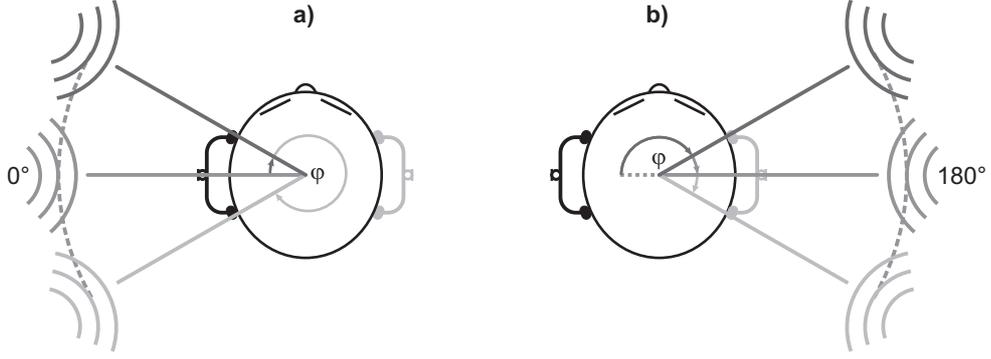


FIG. 3: Noise angles  $\varphi$  for the left ear-cup. a) Angles around  $0^\circ$ . b) Angles around  $180^\circ$ .

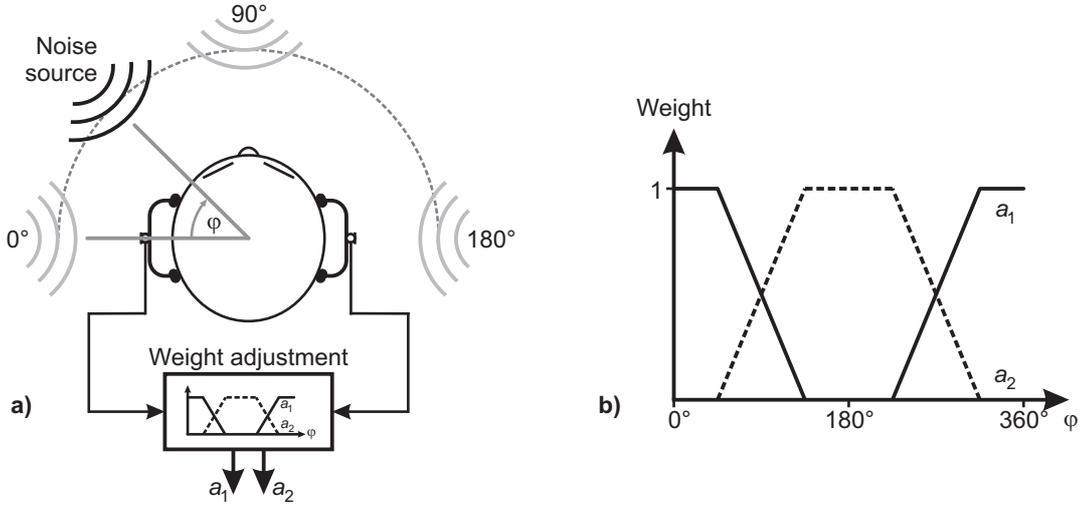


FIG. 4: a) Weights are adjusted proportionally to the noise angle  $\varphi$  which is obtained through an approximation based on the signal power of the reference signals. b) Enlarged weighting function.

### III. NOISE CONTROLLER DESIGN

As explained in the previous section, different problems regarding the use of non-adaptive feedforward controllers in headsets exist. In circum aural headsets, these problems commonly make the use of simple non-adaptive feedforward controllers impractical or inefficient. In the following, a direction dependent feedforward controller (DDFC) in combination with a non-adaptive feedback controller is suggested to resolve the described problems.

### *Direction Dependent Feedforward Control (DDFC)*

In general, the system plants of ANR-headsets are subject to permanent variations. Using solely one non-adaptive feedforward controller, which is designed for noise angles around  $0^\circ$ , results in reduced noise attenuation performance when the angle changes. Figure 3 illustrates situations where the noise source moves and thus the noise angle changes. Measurements showed: In case of noise angles around  $0^\circ$ , refer to Fig. 3a, only marginal deterioration of the attenuation performance is expected. However, in case of noise angles around  $180^\circ$ , as depicted in Fig. 3b, the disturbing noise is even amplified in a narrow frequency range between 350Hz and 1000Hz. This problem can be resolved using a second non-adaptive feedforward controller which is designed for this unfavorable situation of noise angles around  $180^\circ$ . Hence, depending on the noise angle, the control algorithm has to switch between the two different controllers. The algorithm uses one controller for noise angles around  $0^\circ$  and the other controller for noise angles around  $180^\circ$ . Since switching between the controllers results in confusing and annoying acoustical artifacts, a smooth transition between the controllers is implemented. Hereby, the smoothing is accomplished using weighting coefficients which can take values between zero and one. According to Fig. 4a, the weights are adjusted depending on the noise angle and consequently the smoothed controller output is provided by a weighted sum of the two non-adaptive controllers:

$$y_{ff}(n) = y_1(n)a_1(\varphi) + y_2(n)a_2(\varphi). \quad (7)$$

As shown in Fig. 4b, the two weights  $a_1$  and  $a_2$  are adjusted depending on the noise angle  $\varphi$ . The estimation of the noise angle  $\varphi$  is based on the assumption that the reference microphone of the far side ear-cup is obstructed by the head (situation Fig. 2b). Thus, the signal power of this reference microphone is reduced in comparison to the signal power of the ear-cup which is directly exposed to the noise source. Consequently, the estimation of the noise angle can be accomplished using the ratio of the signal power of both reference microphones. This power ratio is proportional to the noise angle  $\varphi$  and thus permits the adjustment of the weights  $a_1$  and  $a_2$ . In order to find a suitable weight combination  $(a_1; a_2)$  for the noise angles, different experiments where the headset is exposed to the noise under different noise angles are accomplished. For each noise angle the weights are adjusted manually until the best weight combination is found. This experiment leads to the approximation of the weighting function as shown in Fig. 4b. It turned out that in case of angles between  $135^\circ$  and  $225^\circ$

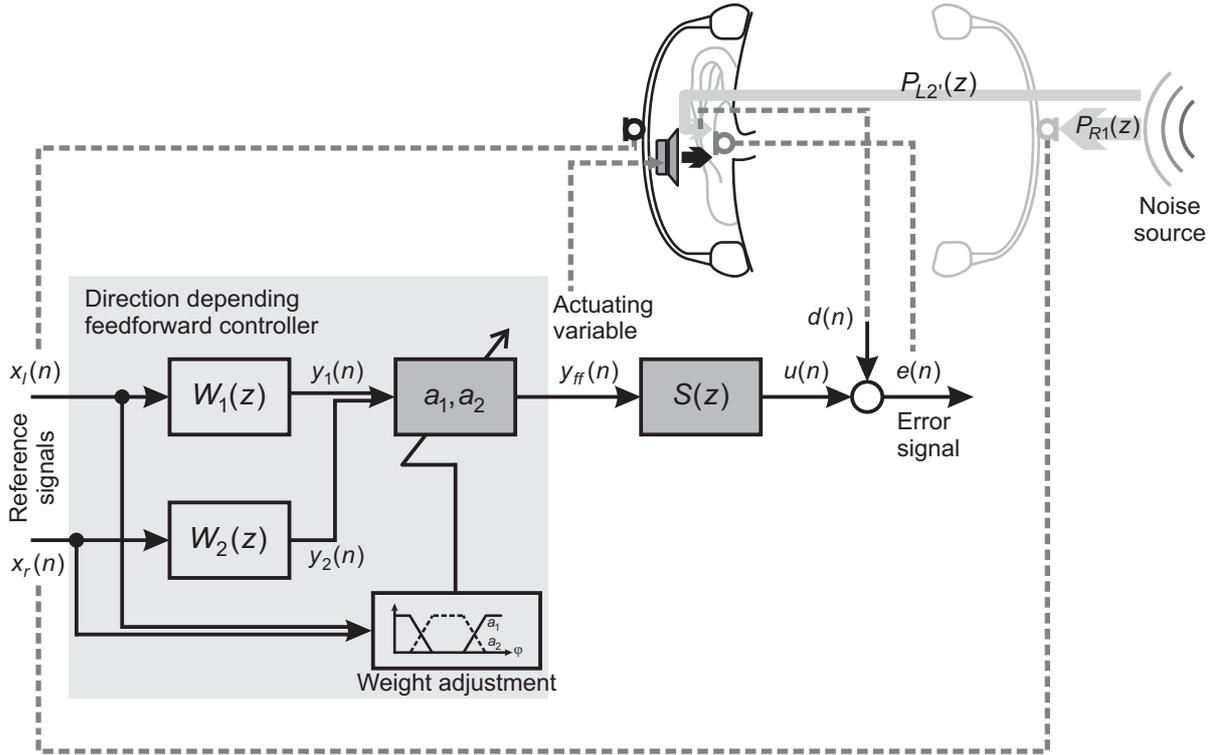


FIG. 5: Direction dependent controller exposed to the noise source under a noise angle of  $180^\circ$ . Remark: Here, the error microphone is only used to measure the residual error signal.

as well as between  $315^\circ$  and  $45^\circ$  the best noise reduction performance is obtained by the introduction of a flat region in the weighting function. The resulting direction dependent weighting is similar to a variable structure controller but with smooth transition rather than abrupt switching. The block diagram of the feedforward control structure is illustrated in Fig. 5.

The direction dependent control approach avoids unfavorable noise amplifications in case of noise angles around  $180^\circ$ . Additionally, due to the smooth transition, artifacts caused by the controller switching are avoided.

As explained previously, the design of two controllers is needed: One has to be optimal for noise angles around  $0^\circ$  and the other for noise angles around  $180^\circ$ . The controller design for noise angles around  $0^\circ$  result in a causal transfer function  $P_{L2}(z)/P_{L1}(z)$  and thus in the feedforward controller:

$$W_1(z) = -\frac{P_{L2}(z)}{P_{L1}(z)S(z)}. \quad (8)$$

In contrast, for noise around  $180^\circ$ , the term  $P_{L2'}(z)/P_{L1'}(z)$  is non-causal. This problem can be resolved using the reference microphone of the opposite ear-cup<sup>4</sup>, see Fig. 5. Thus, the feedforward controller for noise angles around  $180^\circ$  can be expressed as:

$$W_2(z) = -\frac{P_{L2'}(z)}{P_{R1}(z)S(z)}. \quad (9)$$

In comparison with Eq. 6, the delay  $T_1$  becomes greater than  $T_2$  and thus the expression  $P_{L2'}(z)/P_{R1}(z)$  is causal. Hence, the controller of Eq. 9 promises enhanced attenuation performance compared to the controller of Eq. 6.

It has to be mentioned, that the previously stated assumption for the secondary path  $S(z) = 1$ , made in Eq. 6, is not valid in real ANR-systems. Furthermore, acoustic wave propagation results in an unavoidable dead time which introduces non-causality when  $S(z)$  is inverted. However, generally this non-causality is not problematic since it is compensated by the time delay of the expression  $\frac{P_{L2'}(z)}{P_{R1}(z)}$ .

#### *Combined non-adaptive Feedback-Feedforward Control*

In many applications low frequency noise dominates the power spectrum of the disturbing noise. In order to reduce these dominant components effectively, an additional feedback controller is linked to the feedforward controller (Fig. 6). Due to the fact that the secondary path  $S(z)$  is assumed to be invariant during operation as well as the controller is independent of the primary paths, a non-adaptive feedback controller design is chosen.

In the following, it is shown that the linking can be accomplished without introducing negative mutual interactions of the controllers.

Referring to the standard control loop, the controller is designed in terms of a minimum residual error signal  $e(n)$ .

According to the transfer function

$$\frac{E(z)}{D(z)} = \frac{1}{1 + S(z)R(z)} \quad (10)$$

the minimization of the error signal is accomplished by maximizing the magnitude of the denominator  $|1 + R(z)S(z)|$  while maintaining robust stability<sup>26</sup>.

When linking the feedforward controller to the standard feedback control loop it has to be guaranteed that negative effects are excluded. In this context, negative effects refer to

significant deterioration of the noise reduction performance of each single controller. In the following it is shown, that the linking of the two control strategies is feasible without negative mutual interaction.

Referring to the combined control strategy as depicted in Fig. 6, the z-transform of the error signal  $e(n)$  can be written as

$$E(z) = D(z) + U(z). \quad (11)$$

Without loss of generality, it is assumed that exclusively one feedforward controller ( $a_1 = 1, a_2 = 0$ ) is used. Thus, expressing  $E(z)$  in terms of the noise signal's z-transform gives:

$$E(z) = \frac{P_{L2}(z)}{1 + R(z)S(z)}N(z) + \frac{P_{L1}(z)W_1(z)S(z)}{1 + R(z)S(z)}N(z) \quad (12)$$

$$E(z) = [P_{L2}(z) + P_{L1}(z)W_1(z)S(z)] \cdot \frac{1}{1 + R(z)S(z)} \cdot N(z). \quad (13)$$

In order to achieve effective noise reduction, the error signal has to be minimized:

$$E(z) \rightarrow \min. \quad (14)$$

According to Eq. 13 the residual error  $E(z)$  is reduced by both controllers: While the first term of expression 13 is decrease by the feedforward controller, the second (the fraction) is reduced by the standard feedback control loop. Hence, the combined control strategy leads to improved noise reduction performance and with respect to Fig. 6, the entire control law of the combined control approach becomes:

$$y(n) = \mathcal{Z}^{-1}\{X_L(z)W_1(z)a_1 + X_R(z)W_2(z)a_2 - R(z)E(z)\}. \quad (15)$$

#### IV. ATTENUATION PERFORMANCE

The direction dependent controller is realized with two fourth order feedforward transfer functions  $W_1(z)$  and  $W_2(z)$ , whereas the feedback controller implements 11 poles and 11 zeros.

The measurement setup consists of a loudspeaker representing the noise source and a self constructed artificial head with an ear simulator reproducing the human's ear acoustics. As the acoustical front end, the series product *Sennheiser HMEC 350* is used. The experiments

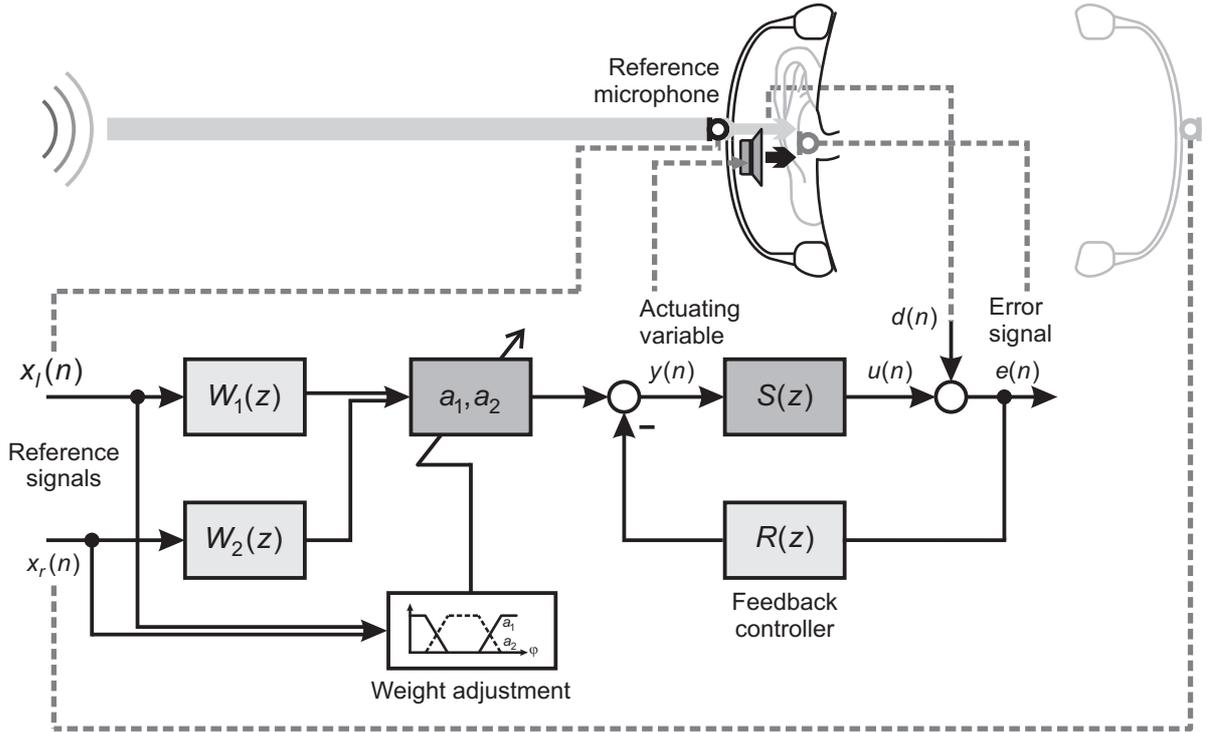


FIG. 6: Entire control diagram consisting of the direction dependent feedforward controller linked to a non-adaptive feedback control loop. Remark: Noise exposure is exemplarily accomplished under a noise angle of  $0^\circ$ . In general, this angle can be arbitrary.

are accomplished using a band limited white noise and an ear-cup leakage that is approximately equivalent with an ear-cup leakage when the headset is borne by a person.

The first measurements show the advantage of the proposed direction dependent feedforward approach compared to a conventional non-adaptive feedforward controller. The second measurements illustrate the attenuation performance of the entire control strategy according to Fig. 6. The noise attenuation performance is compared to a series product as well as an adaptive ANR-system.

### *Feedforward Noise Reduction*

Figure 7 compares the direction dependent feedforward active noise attenuation according to Fig. 5 with the feedforward noise reduction performance of a non-adaptive feedforward controller. Two noise situations are examined for both controllers. The left part of Fig. 7 illustrates the attenuation for a noise angle of  $0^\circ$  which is the noise angle which the

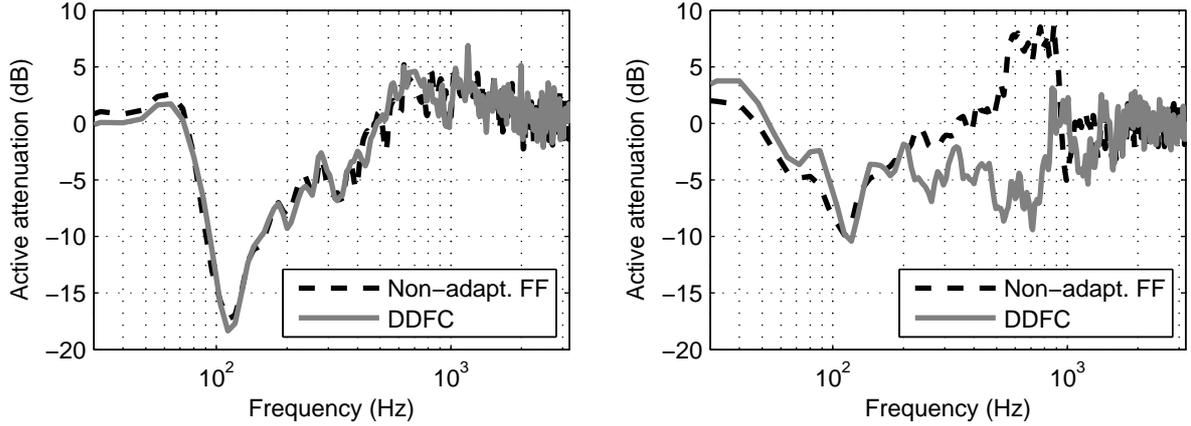


FIG. 7: Active noise attenuation of the direction dependent feedforward controller (grey) compared to a conventional non-adaptive feedforward controller (dashed black). Left: Noise angle  $0^\circ$ . Right: Noise angle  $180^\circ$

non-adaptive feedforward controller is designed for. In case of the direction dependent feedforward controller (DDFC), a noise angle of  $0^\circ$  leads to the weight-pair  $a_1 = 1, a_2 = 0$ . Thus, solely the controller  $W_1(z)$  is used. As expected, the two control strategies result in almost equal attenuation performance.

The right part of Fig. 7 depicts the unfavorable noise angle of  $180^\circ$ . The direction dependent algorithm adjusts the weights to  $a_1 = 0$  and  $a_2 = 1$ . Thus, only  $W_2(z)$  is used and a better attenuation performance compared to the non-adaptive feedforward controller is obtained. Figure 8 clarifies the difference of the attenuation performance. Here, the quotient

$$\frac{\text{DDFC active noise reduction}}{\text{Non-adaptive FF active noise reduction}}$$

is diagramed. For a noise angle of  $180^\circ$ , the direction dependent approach shows better attenuation performance in the frequency range between 200Hz and 900Hz. Especially in the range between 500Hz and 850Hz the direction dependent controller shows more than 10dB better noise attenuation compared to the conventional non-adaptive feedforward controller.

#### *Noise Reduction of the Combined Control Strategy*

The previous illustrates the advantages of the direction dependent approach compared to the common non-adaptive feedforward controller. According to section III, it is possible

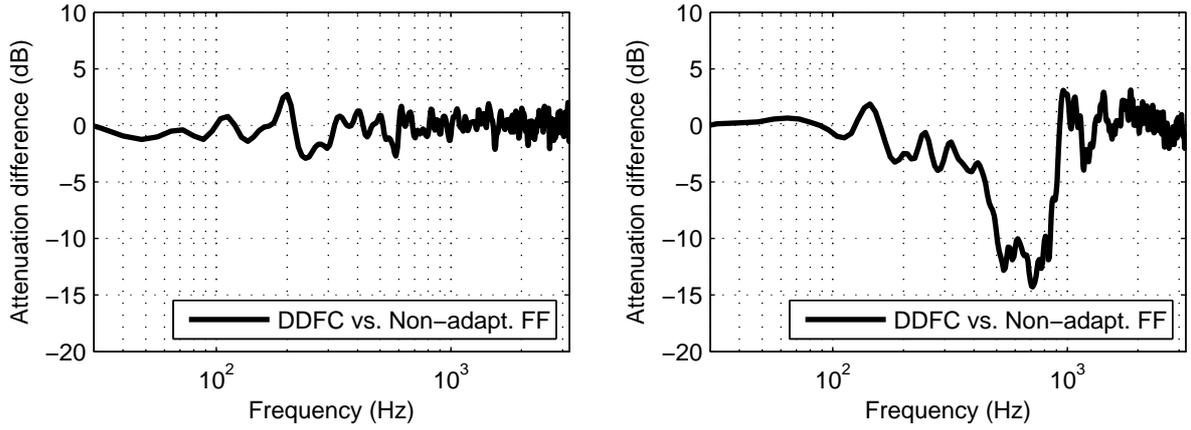


FIG. 8: Relative comparison of the active noise attenuation performance. Left: Noise angle  $0^\circ \rightarrow$  similar attenuation. Right: Noise angle  $180^\circ \rightarrow$  improved attenuation of the direction dependent feedforward approach.

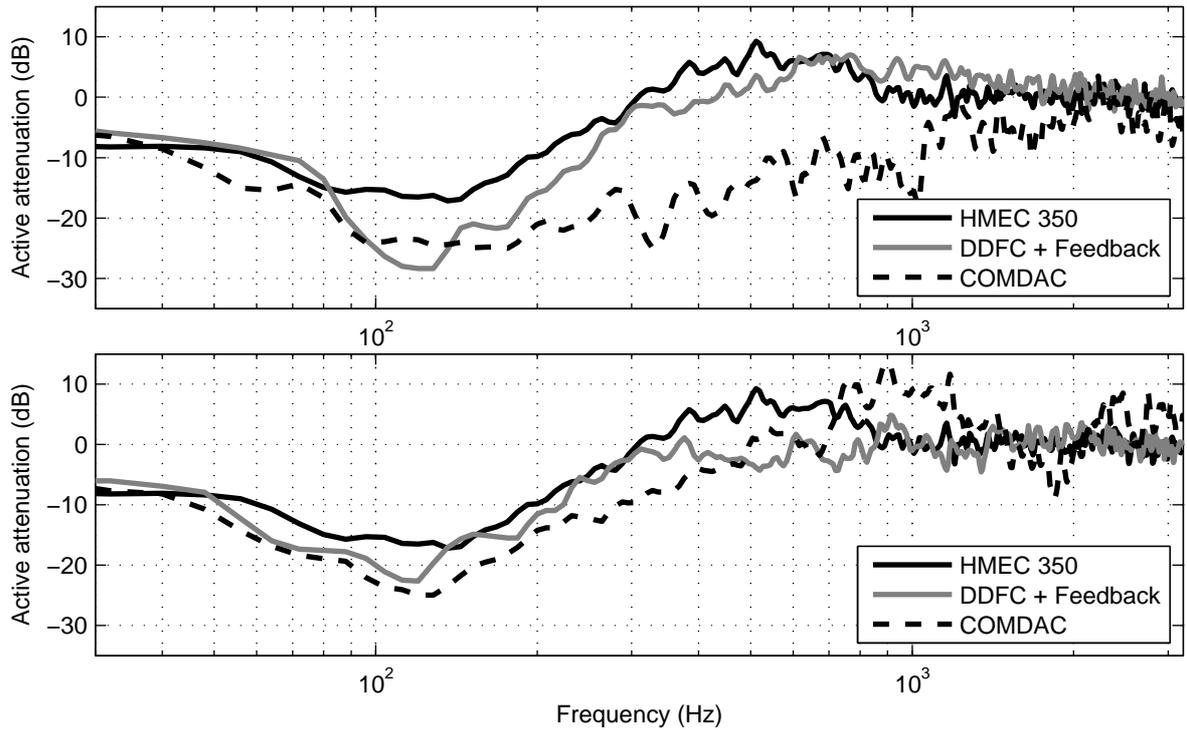


FIG. 9: Comparison of the proposed combined direction dependent approach with other ANR-strategies. Upper part: Noise angle  $0^\circ$ . Lower part: Noise angle  $180^\circ$ .

to combine the direction dependent controller with a feedback loop which leads to a control strategy that shows improved attenuation performance in the low frequency range. Figure 9 compares the attenuation performance of the proposed combined control approach with the series product *HMEC 350* and a prototype which was developed in the framework of a project called *COMDAC*<sup>4</sup>. The headset *HMEC 350* is solely equipped with a non-adaptive feedback controller and the *COMDAC* prototype uses a non-adaptive feedback controller as well as an adaptive feedforward controller with 140 controller coefficients. Thus, the *HMEC 350* forms the most effective ANR-system with least controller complexity. Due to its computational expensive control algorithm, the most complex system is formed by the *COMDAC* prototype. All three ANR-systems base on the same acoustical front-end which make the active attenuation of these systems comparable.

The upper diagram of Fig. 9 shows the active attenuation of the different ANR-headsets for a noise angle of  $0^\circ$ . Due to its complexity, the *COMDAC* prototype outperforms the other ANR-systems. However, in the frequency range from 80Hz to 200Hz the combined direction dependent controller results in about 10dB better attenuation than the *HMEC 350* for almost the same amount of complexity.

In case of a noise angle of  $180^\circ$  as depicted in the lower part of Fig. 9, solely the combined direction dependent approach shows no noise gain at frequencies higher than 350Hz. It has to be remarked, that the active attenuation performance of the adaptive *COMDAC* prototype is significantly dependent on the spectrum of the disturbing noise. This means, that in case of a narrow band noise spectrum, the noise reduction performance in the according frequency range will be further enhanced compared to Fig. 9.

## V. CONCLUSION

The proposed control approach consists of a direction dependent feedforward controller (DDFC) in combination with a non-adaptive feedback controller. The direction dependent controller is able to switch between two different feedforward controllers. In case of varying system plants, this result in improved noise reduction compared to the conventional non-adaptive feedforward approach. In order to avoid acoustical artifacts which are linked to the switching, a smoothed transition between the controllers is accomplished. The direction dependent feedforward controller is linked to a non-adaptive feedback control loop, which

results in improved broadband noise attenuation performance. Compared to an adaptive broadband ANR-system, the proposed control approach guarantees a significant reduction of the computational effort.

---

\* Electronic address: `jens.graf@imr.uni-hannover.de`

† Electronic address: `eduard.reithmeier@imr.uni-hannover.de`

## VI. LITERATURE

- <sup>1</sup> Elliott, S.J., 2001, "Signal processing for active control", Academic Press, San Diego.
- <sup>2</sup> Kuo, S.M., Morgan, D.R., 1996, "Active Noise Control Systems - Algorithms and DSP Implementations", Wiley-Interscience Publication, New York.
- <sup>3</sup> Wolter, B., Peissig, J., Foudhaili, H., Reithmeier, E., 2007, "Combined feedback and adaptive feedforward active noise control in headsets", Proc. 36th International Congress and Exhibition on Noise Control Engineering, Internoise, Istanbul.
- <sup>4</sup> Foudhaili, H., 2008, "Kombinierte Feedback- und adaptive Feedforward-Regelung fuer die aktive Laermreduktion in einem Kommunikations-Headset", Shaker Verlag, Aachen.
- <sup>5</sup> Rafaely, B., 2001, "Active noise reducing headset an overview", Proc. 33th International Congress and Exhibition on Noise Control Engineering, Internoise, The Hague.
- <sup>6</sup> Reithmeier, E., Graf, J., 2008, "Combination of non-Adaptive and Adaptive Control Strategies for Active Noise Reduction in Headsets", Proc. 9th International Conference on Motion and Vibration Control, Movic, Munich.

## Nomenclature and Abbreviations

$a_1$	Weight of the first feedforward controller
$a_2$	Weight of the second feedforward controller
$d(n)$	Primary noise measured by the error microphone
$D(z)$	Z-transform of the primary noise
$e(n)$	Error signal / residual error
$E(z)$	Z-transform of the error signal
$n(n)$	Noise source signal
$N(z)$	Z-transform of the noise source signal
$P_a(z)$	Causal transfer function
$P_b(z)$	Causal transfer function
$P_{L1}(z)$	Transfer function from a left-hand noise source to the left reference microphone
$P_{L2}(z)$	Transfer function from a left-hand noise source to the left error microphone
$P_{L1'}(z)$	Transfer function from a right-hand noise source to the left reference microphone
$P_{L2'}(z)$	Transfer function from a right-hand noise source to the left error microphone
$S(z)$	Secondary path from the compensation loudspeaker to the error microphone
$T_1(z)$	Time delay of the transfer function $P_{L1'}(z)$
$T_2(z)$	Time delay of the transfer function $P_{L2'}(z)$
$u(n)$	Antinoise generated by the loudspeaker
$W_1(z)$	First feedforward controller
$W_2(z)$	Second feedforward controller
$x_l(n)$	Reference signal of the left ear-cup
$x_r(n)$	Reference signal of the right ear-cup
$X_L(z)$	Z-transform of the left reference signal
$X_R(z)$	Z-transform of the right reference signal

$y(n)$	Actuating variable
$y_1(n)$	Output of the first feedforward controller
$y_2(n)$	Output of the second feedforward controller
$y_{ff}(n)$	Actuating variable of the DDFC
$Y_{ff}(z)$	Z-transform of the DDFC actuating variable
$\mathcal{Z}\{\cdot\}$	Z-transform operator
$\varphi$	Noise angle

### Abbreviations

ANR	Active Noise Reduction
DDFC	Direction dependent feedforward control
FF	Feedforward controller